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The Role of Expectations in Commodity Price Dynamics:
Evidence from Oil Data

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The Role of Market Expectations in Commodity Price Dynamics: Evidence from Oil Data[*]

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Abstract This paper examines the contribution of market expectations to commodity price dynamics. It proposes a dynamic competitive storage framework with an expectations shock explicitly along with concurrent shocks to study the commodity price movements. This allows for a more refined analysis of the expectations' effect on price and inventory and the estimation of the expectations. Applied to the world crude oil market, it finds that the contribution of market expectations to the crude oil spot price movements is limited from 1987 to 2014.

Keywords: commodity spot price; commodity inventory; expectations shock; dynamic equilibrium model; state space model

JEL Classification Numbers: C32, G18, Q38, Q41, Q48.

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1 Introduction

Inventory behavior is usually linked to the expectations about the future. In the discussion of the causes of the recent crude oil price increases especially during 2007-2008, one key question is whether speculation played an important role. Regardless of their stand on it, researchers turn to inventory data for a better understanding of speculative or precautionary incentive in the oil market, as anticipation of future increases in oil price could lead to speculative inventory increase and result in immediate price increase.[1] Earlier work like Brennan (1958) has already pointed out that inventory is related to the expected change in price. Applying this intuition in the oil context, Hamilton (2009b) proposes a link between speculation and the inventory movements. Empirical studies like Kilian and Murphy (2014) and Knittel and Pindyck (2016) argue against a major contribution of speculation where the authors identify the forward-looking element of the real price with data on oil inventories. However, Juvenal and Petrella (2014) find a more important role of speculation also using data on inventories.

To avoid ambiguity, this paper uses a neutral term "expectation" and defines it mathematically. Building "expectation" in a rational expectation equilibrium model, this paper specifically focuses on the difference between shocks to market expectations and shocks to contemporaneous market condition. It contributes in two ways to the literature on commodity price dynamics, especially the discussion on the role of speculation. Theoretically, the model solution provides new insights of the features of market expectations' effect on price and inventory. Empirically, it estimates a structural model using oil market data to quantify the contribution of market expectations.

The new insight from the structural model is the dynamic shape of the expectations' effect. Everything else being equal, an expectations shock leads to a larger change in the expected future price than in the spot price, while a contemporaneous shock's effect

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[1] The term "expectations" as discussed in this paper will be defined on page 3.

is the other way around. As a result, an expectations shock would not only result in different immediate changes, but also response functions shaped differently over time compared to a contemporaneous shock. This theoretical knowledge enables more refined identification of market expectations in the empirical analysis.

The intuition works as follows. Today's expectations of a strong future demand relative to supply will result in a higher spot price today, due to the lower current availability of the commodity from the accumulated inventory in response to such expectations. This immediate effect has been discussed in earlier literature. Furthermore, while the inventory accumulation smooths the expected quantity (demand/supply) fluctuations, it would be too costly to accumulate inventory so much that the price does not change or changes little on the future date when the strong demand actually hits. Thus the increase in the expected future price would be larger than that in the spot price. The resulting price response function is hump-shaped.

On the other hand, today's strong relative demand to supply will also result in a higher spot price, as discussed in earlier literature. Furthermore, it will also instantaneously result in a higher expected future price due to lower future availability from the depleted inventory (everything else being equal). However, the impact of today's strong demand dissipates, thus the increase in the expected future price is smaller than that in the spot price. The resulting price response function is monotonically decreasing after the initial jump.

This refined intuition can be captured by an "expectations shock" which has no contemporaneous but only lagged impact on the supply and demand. Here the "expectations" specifically refers to the innovations and macroeconomic activities that could affect the commodity market supply and demand with a delay, in the style of the news shock that has been discussed by Beaudry and Portier (2006) and adopted by a large macroeconomic (DSGE) literature like Davis (2007), Barsky and Sims (2011),

Jaimovich and Rebelo (2009) and others.

More specifically, the idea is that agents in the market may learn about some production capacity that has been recently installed and will be implemented in the future, at which time they expect the supply to rise. Similarly, agents could learn that a commodity will be utilized with higher efficiency in the future at which time they expect the demand to shift. Such expectations have no effect on the current market supply and demand condition, but do affect agents' current inventory decision, the spot and expected future prices. It is such expectations that are referred to as the "expectations" in the model.

In addition to the dynamic shape, the analysis illustrates the key importance of the price elasticity of demand in the price dynamics, extending the views of Hamilton (2009b), Baumeister and Peersman (2013) and Kilian and Murphy (2014). This paper finds that the less elastic the demand, the larger the price and inventory responses to changes in the market condition, everything else being equal.

The structural model also makes it straightforward to utilize the futures market data in the empirical application. Recent theoretical work like Sockin and Xiong (2015) highlights the informational feedback effects of commodity futures prices. Cheng and Xiong (2014) aruge that relying on only the inventory data for identifying effects of speculation ignores the futures prices which reflect agents' expectations. In the empirical application of this model, both inventory and futures market data have been used to identify market expectations.

To the best knowledge of the author, this paper is the first to quantify the effect of expectations by solving and estimating a structural model and introducing a mathematical definition of "expectations". The structural framework allows for the precise mapping of mathematical expression to economic interpretation, and thus the refined identification with the additional dynamic shape feature of the expectations versus the

contemporaneous shocks. This is different from earlier empirical work like Kilian and Murphy (2014), Juvenal and Petrella (2014), and Baumeister and Hamilton (2015). Beidas-strom and Pescatori (2014) discusses the dynamic dimension, however argues instead the price effect of the speculative demand shock is "monotonically declining" after the initial period. Knittel and Pindyck (2016) constructs an analytical framework for a storable commodity, but the model is not solved dynamically. In terms of the modelling and empirical methodology, Unalmis et al. (2012) is the mostly closely-related. They incorporate oil storage into a DSGE model, but does not contain an expectation component and cannot comment on the cumulative contribution of expectations to the price movements.

This model differs from one strand of earlier storage and price dynamics literature like Wright and Williams (1982, 1984) and Deaton and Laroque (1992, 1995, 1996) and the more recent Dvir and Rogoff (2010) and Arseneau and Leduc (2013) in modelling inventory stock-out. Instead, observing that oil market does not typically experience stock-outs, this paper models a non-linear marginal convenience yield function as in Pindyck (1994) such that when the inventory approaches zero, the marginal convenience yield approaches infinity. Intuitively this setting implies that it is always beneficial to hold inventory. As a result the inventory will always stay positive.[2]

The empirical results using crude oil spot and futures prices and inventory data show that under conventional assumption of the price elasticity of demand, the market expectations have contributed little to the crude oil spot price movements from 1987 to 2014. The market fundamentals are the main drivers of the price movements. This confirms the results of earlier literature studying the role of speculation based on the theory of storage.

The paper is planned as follows. Section 2 introduces the model. Section 3 solves

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[2]Similarly, Eichenbaum (1984) argues for the technological reason in addition to the speculative motive for voluntarily-held inventory.

the model, and discusses the theoretical implications on the price-inventory dynamics in an equilibrium model under rational expectation. Section 4 presents the estimation results and the discussion of the role of the shocks during the past price and inventory movements. Section 5 concludes.

2 The Model

This section sets up the model for oil market equilibrium with inventory. Although it has been interpreted in the oil market context, the model can be generally applied to most storable commodity markets in which no stock-out has been observed. In this model of the world oil market, the price is determined by the oil supply and demand. The quantities supplied and demanded are not necessarily the same, as the market also has demand for inventory, based on the current market and the expectations of the future.

2.1 Oil Price Determination

Starting with a general inverse demand function for crude oil, the oil price P_t is determined by the oil consumption Q_t^d, and a measure of overall economic performance Y_t^d. Specifically, Y_t^d captures the shifts of the demand curve driven by the global economic fluctuations. For example, Kilian (2009) has argued that the demand for industrial raw materials has been fuelled by the emerging economies in Asia such as China and India after 2002.

Furthermore, this paper posits this inverse demand function to be homogeneous of degree zero, i.e. only the consumption relative to the overall economic performance matters, as oil consumption and world economic performance are highly correlated.

Thus a CES inverse demand function can be used:

$$P_t = c\left(\frac{Q_t^d}{Y_t^d}\right)^{-\frac{1}{\gamma}} \tag{1}$$

where c is a scalar and γ measures the price elasticity of demand. This inverse demand function is decreasing in Q_t^d and increasing in Y_t^d.

Denoting the available inventory at the beginning of period t by N_t, and the inventory held for next period $t+1$ by N_{t+1}, the crude oil consumption Q_t^d equals to the crude oil production Q_t^s less the change in inventory $N_{t+1} - N_t$ in the market equilibrium:

$$P_t = c\left(\frac{N_t + Q_t^s - N_{t+1}}{Y_t^d}\right)^{-\frac{1}{\gamma}} \tag{2}$$

2.2 Inventory Decision

In addition, the demand for inventory-holding arises from the uncertainty about the future. A profit-maximizing oil producer (or buyer) in a competitive market makes decision with regards to its inventory-holding following the first-order condition when the inventory is positive[3]:

$$P_t = \beta E_t[P_{t+1}] - E_t[MIC_{t+1}] \qquad \text{if} \quad N_{t+1} > 0 \tag{3}$$

where MIC is the net marginal cost of holding inventory, which includes the physical cost of storage as well as the convenience of storage (see Brennan (1958) and others). Whenever positive inventory is held, an optimal inventory decision N_{t+1} at time t would be such that the resulting net marginal cost of holding inventory $E_t[MIC_{t+1}]$ would be just covered by the marginal revenue, or the expected intertemporal price change

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[3]This first-order condition is the same regardless of whether it is the producer or the buyer holding the inventory.

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$\beta E_t[P_{t+1}] - P_t$.

Since in the commodity market, zero inventory is rarely observed, the net marginal cost of holding inventory is modelled such that the optimal N_{t+1} would always be positive. To achieve this it is assumed that the net marginal cost converges to negative infinity when inventory is drawn down to near zero. Thus, even when the price is expected to fall and the expected intertemporal price change $\beta E_t[P_{t+1}] - P_t$ is very negative, the inventory still won't be drawn out completely. Intuitively, inventory facilitates production and delivery scheduling and avoids stock-outs in the face of fluctuating demand and changing supply technology. These benefits motivate producers to hold inventory even if they expect the price to fall, as discussed in Brennan (1958). The exponential function for the net marginal cost of holding inventory as suggested by Pindyck (1994) has been adopted, assuming that there is a constant marginal inventory-holding cost δ, and that the net marginal cost is affected positively by the current price as well as the relative inventory held.[4] Furthermore an inventory adjustment cost is introduced, following earlier literature like Eichenbaum (1984), observing that the relative inventory data (the inventory held relative to the quantity demanded) is much less volatile compared to the price even after removing the seasonality.

$$MIC_{t+1} = P_t * [\delta + \alpha(\frac{N_{t+1}}{N_{t+1} + Q_{t+1}^s - N_{t+2}})^{-\phi} + \Theta(\frac{N_{t+1}}{N_t}) - \beta * \Theta(\frac{N_{t+2}}{N_{t+1}})] \quad (4)$$

The net marginal cost of storage here takes into consideration the physical cost of holding inventory δ, the intangible benefit of inventory-holding to avoid stock-out (the exponential part with $\alpha < 0$) and the inventory adjustment costs Θ (which is a function of relative inventory changes) for both the current and next periods. The exponential

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[4]Pindyck (1994) refers to the negative net marginal cost of storage as "the net marginal convenience yield", and proposes an exponential form for the latter based on the observation that the scatter plot of relative inventory against the net marginal cost of storage is nonlinear.

part captures the intangible benefit of inventory-holding in a way such that the benefit would be low when the inventory level is already high relative to the quantity demanded, and vice versa. Such setting guarantees that the optimal inventory level is never drawn down to zero. Θ is assumed to be zero when there's no change in inventory, and to have constant marginal adjustment cost (Θ'). More detailed discussion of the parameters and the functions will be available in later section of the model solution and its estimation.

2.3 Exogenous Shocks in the Model: Modelling Expectation

The key part is modelling the factors driving the price and inventory, including contemporaneous and expectations factors. The model itself does not attempt to explain how demand, supply and the expectations about them arise, and thus treat them as exogenous.

On the supply side of the market, the log of world crude oil supply can be reasonably assumed to follow a random walk process with a drift.[5]

$$\log(Q_t^s) = \log(Q_{t-1}^s) + \log(\mu_t^s) \tag{5}$$

$$\log(\mu_t^s) = \bar{\mu} + \epsilon_t^\mu \sim N(0, \sigma_\mu^2) \tag{6}$$

The process for the demand side is modelled implicitly. The demand shifter, or the process for overall economic activities Y_t^d, can be thought of as some function of either world GDP or industrial production index as discussed earlier. Regardless which one of these measures best approximates Y_t^d, the process is quite possibly non-stationary. However, in the oil/commodity market context, it is also reasonable to think that the overall economic activities are overall balanced with the supply in the long run, as strong economic activities encourage new production capacity instalment

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[5]Figure 4 and 5 in the empirical section provide more evidence: the log of world crude oil supply contains a random walk, and its first difference is stationary.

and new exploration, and weak economic activities lead to fewer drilling activities. This stationary assumption is especially important for solving the model, and will be discussed in next subsection.

Thus, instead of modelling Y_t^d explicitly as another random walk, the stationary relative supply $\frac{Q_t^s}{Y_t^d}$ is modelled:

$$\log \frac{Q_t^s}{Y_t^d} = y_t^\tau + y_t^c \tag{7}$$

$$y_t^\tau = \rho^\tau y_{t-1}^\tau + n_{t-1}^\tau + \epsilon_t^{y_\tau} \qquad\qquad \epsilon_t^{y_\tau} \sim N(0, \sigma_{y_\tau}^2) \tag{8}$$

$$y_t^c = \rho^c y_{t-1}^c + \epsilon_t^{y_c} \qquad\qquad \epsilon_t^{y_c} \sim N(0, \sigma_{y_c}^2) \tag{9}$$

$$n_t^\tau = \rho^{n_\tau} n_{t-1}^\tau + \epsilon_t^{n_\tau} \qquad\qquad \epsilon_t^{n_\tau} \sim N(0, \sigma_{n_\tau}^2) \tag{10}$$

Here expectation is introduced. The relative supply process contains two types of components: contemporaneous and forward-looking. The contemporaneous components are the persistent y_t^τ and the temporary y_t^c, and both are AR(1) processes, with $\rho^\tau > \rho^c$. The expectation n_t^τ is modelled as an AR(1) process with autoregression coefficient ρ^{n_τ}.

The expectation n_t^τ is modelled similarly to the news in the DSGE literature. It captures the events that could affect the market demand and supply with delay as Equation 8 shows.[6] When the market expectations at t changes, even though the relative supply in the current period t is not affected, rational market participants would still respond immediately to the expectation change by adjusting inventory, which results in a contemporaneous price change. This expectation in the model captures the forward-looking component of price determination in the market: if the market agents

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[6]Note the expectation n_t^τ is modelled to affect the relative supply $\frac{Q_t^s}{Y_t^d}$ via y_t^τ rather than directly. This is because the knowledge of a future event might be acquired several periods in advance. Such parsimonious setting allows more versatile dynamics in capturing the market expectations, so that the actual peak change in the relative supply takes place several periods afterwards, despite that n_t^τ affects it with a fixed one-period lag. See Figure 1 and Figure 2 in the simulation for the illustration.

believe that the price would be higher in the future, such expectations would drive up the price and inventory today.

It is worth noting that the above assumptions view the supply as exogenous to the demand while the two remain cointegrated. The assumption that the supply shock ϵ_t^μ is independent of the shocks ($\epsilon_t^{y\tau}$, ϵ_t^{yc} and $\epsilon_t^{n\tau}$) to the cointegration relationship ($\log \frac{Q_t^s}{Y_t^d}$) implies that the supply is not affected by the demand side. This is in line with the empirical findings that the demand side shocks do not affect the supply (see Hurn and Wright (1994), Mauritzen (2016) and Anderson et al. (to appear)), but sharply contrasts with the identification restrictions of Kilian and Murphy (2014) and Juvenal and Petrella (2014).

2.4 Model Overview and Equilibrium

Normalization of some variables is necessary in order to solve for the steady state of the model and the equilibrium path since they contain trends (Q_t^s, Y_t^d). Following the macroeconomic literature in treating the variables with a trend, they are normalized by the world supply. The stationarity assumption on $\frac{Q_t^s}{Y_t^d}$ discussed earlier guarantees that the model has a steady state.

Such normalization of variables in Equation 2 results in the "relative supply" $\frac{Q_t^s}{Y_t^d}$, which will be denoted by a lower-case letter, $q_t^s = \frac{Q_t^s}{Y_t^d}$, and the "effective inventory" level, $n_{t+1} = \frac{N_{t+1}}{Q_t^s}$. Note that the "relative supply" q_t^s is assumed to be stationary (see Equation 7 to 10), thus the model has a steady state.

Equation 2 then can be rewritten in terms of the "effective inventory" n_t and the "relative supply" q_t^s:

$$P_t = c[(n_t/\mu_t^s + 1 - n_{t+1}) * q_t^s]^{-\frac{1}{\gamma}} \tag{11}$$

Similarly, the normalization of variables in Equation 3 and 4 results in the equations

rewritten as:

$$P_t = \beta E_t[P_{t+1}] - E_t[MIC_{t+1}] \qquad (12)$$

$$MIC_{t+1} = P_t * [\alpha(\frac{n_{t+1}/\mu_{t+1}^s}{n_{t+1}/\mu_{t+1}^s + 1 - n_{t+2}})^{-\phi} + \delta + \Theta(\frac{n_{t+1}}{n_t/\mu_t^s}) - \beta * \Theta(\frac{n_{t+2}}{n_{t+1}/\mu_{t+1}^s})] \quad (13)$$

where $\mu_{t+1}^s = \frac{Q_{t+1}^s}{Q_t^s}$, as defined in Equation 5.[7]

Now the full model is written in the normalized term as Equations 11, 12 and 13, along with the exogenous processes μ_t^s, y_t^τ, y_t^c and n_t^τ given by Equations 6 7 8 9 10.

The equilibrium path is defined as follows: taking as given the exogenous processes μ_t^s, y_t^τ, y_t^c, n_t^τ and the resulting q_t^s, and an initial stock of effective inventory n_0, the equilibrium of the model is a sequence of $\{P_t, n_{t+1}\}$ that satisfies the optimality conditions of inventory-holding (Equations 12 and 13) and the market clearing condition (Euquation11).

3   Solving the Model

The solved equilibrium price and inventory are functions of the current and expected market demand/supply. The model solution is written in a state space form and will be illustrated using simulated impulse response functions of the price and inventory to the underlying shocks. The simulated impulse responses will also be compared to the sign restrictions widely adopted in recent empirical literature epitomized by Kilian and Murphy (2014).

The illustration shows that "expectation" differs from contemporaneous components in more than the immediate price and inventory responses they cause. The dynamic shapes of the responses over time to different shocks also differ. Also, the model solution

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[7]Note that $log(\mu_{t+1}^s)$ is the world supply growth rate.

reveals that the price elasticity of demand, γ, plays a key role in the magnitude of the price and inventory responses. Everything else being equal, the more inelastic the demand, the larger the magnitude of the price and inventory responses to the underlying shocks, especially to the expectations shock. The persistence of the underlying shocks also matters to the magnitude.

3.1 Model Solution

The model is solved as follows: for an arbitrarily parameterized model (the parameters will be estimated in section 4), it is first log-linearized around its deterministic steady state; the resulting linear rational expectations model is then solved as in Blanchard and Kahn (1980).

In the first step, the resulting linearized model has all variables measured in terms of their log deviations from the steady state values. Then, the current-period spot price (P_t) and next-period effective inventory (n_{t+1}) are solved as linear functions of the predetermined current-period effective inventory (n_t) and the realized shocks ($\hat{\mu}_t^s$, y_t^τ, y_t^c, n_t^τ). This solution is written in a state space form with the currently available effective inventory (n_t) and the exogenous shocks ($\hat{\mu}_t^s$, y_t^τ, y_t^c, n_t^τ) as the state variables, and the spot price (P_t) as the observed variable. The expected future spot price ($E_t(P_{t+1})$) could also be attained. Appendix A provides the details of the solution algorithm.

3.2 Simulated Impulse Response Functions

The model solution is illustrated by the simulated impulse response functions. The arbitrary baseline parameterization is summarized in Table 1.

3.2.1 The Mechanism of the "Expectations Shock"

The illustration of the simulated impulse response functions (Figure 1 and 2) shows that the "expectations shock" indeed captures how expectations work. The impulse response functions to the "expectations shock" show zero immediate response of the relative supply, but non-zero immediate response of the price and inventory.

The response of the relative supply is illustrated in Figure 1. Suppose the world supply is constrained, or expected to be constrained. All shocks have been normalized to cause a decrease in the relative supply. Both contemporaneous shocks (persistent y_t^τ and temporary y_t^c) cause a drop immediately in the relative supply, while the expectations shock (n_t^τ) causes zero immediate change, and the drop takes place only from the second period on[8].

The price and inventory responses are compared in Figure 2, first and third columns. When the world supply is constrained or expected to be constrained, the price and inventory would respond as follows. The price immediately jumps under all three shocks. Inventory is also immediately affected, though it is drawn down under the contemporaneous shocks, but accumulated under the expectations shock.

3.2.2 The "Expectations Shock" Has More than the Immediate Effects

The simulated responses are consistent with what the literature uses to identify forward-looking behavior. For example, Kilian and Murphy (2014), Juvenal and Petrella (2014) and Beidas-strom and Pescatori (2014) posit that, the "speculative demand" shock has "a positive impact effect on inventory accompanying a spot price increase", similar to the immediate effect on the spot price and effective inventory dis-

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[8]The exact peak time and the magnitude of the peak effect of the expectations shock depends on the specific parameterization of the stochastic process, thus Figure 1 is only for qualitative illustration in these aspects. The important feature is that when the event is first learned, i.e. "expected", the market fundamentals have not changed yet.

cussed above.[9] However, the structural model of inventory shows more features of the expectations than the immediate effect. Some are different from the literature, some have not been discussed yet.

First, due to the definition of "expectations" in this paper, the expectations shock in this model causes zero immediate changes in the market fundamentals (Figure 1), whereas in the literature, "the speculative demand" is assumed to have a non-zero impact effect on supply and economic activity (or the overall demand).[10] This indicates that the comparison of the estimation results should be careful.

Second, the time path of price after the expectations shock is qualitatively different from that after the contemporaneous ones. While all shocks cause a price increase, the price path after an expectations shock is hump-shaped: it is increasing first, gradually reaching its peak then returning back down (Figure 2, third row). The peak price effect coincides with the peak effect on the relative supply. The price path after a contemporaneous shock is monotonically decreasing: the immediate effect on price is the peak effect (Figure 2, first row). This is also reflected in the positive expected change in price ($E(P_{t+1} - P_t)$) after the expectations shock versus the negative change after the contemporaneous shocks (Figure 2, second column).

Intuitively, under the expectations shock, the relative supply is affected only at the future date, and it is not economic for the immediate inventory accumulation to be larger than the actual future reduction in the relative supply. Thus, the immediate price increase would be smaller than the peak increase caused by the expectations shock. This dimension of the effect of expectations has not been discussed in the literature, and the dynamic response profile has been largely ignored in the identification of the

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[9]See for example the sign restrictions adopted to identify "speculative demand shock" in Kilian and Murphy (2014), Juvenal and Petrella (2014) and Beidas-strom and Pescatori (2014).

[10]To be exact, in this model the expectations shock causes zero impact changes in the "relative supply", since demand and supply are written as a whole and not differentiated. However, in a general equilibrium framework where demand and supply are modelled explicitly, the expectations modelled as in Section 2 would still have zero impact effect on both.

expectations.

Third, the magnitude of the responses depends on the persistence of the shocks. Using the contemporaneous shocks as an example. In terms of the magnitude of the effect, the persistent shock affects price more and inventory less than the temporary shock, other things being equal (Figure 2, first and second rows). Intuitively, when the disruption to relative supply lasts long, there would be relatively less incentive to drawn down inventory by a large amount immediately. Rather, it would be drawing down inventory over a longer period of time, in order to smooth out the disruption in the relative supply.

Last but not least, the magnitude of the impulse responses also greatly depends on the price elasticity of demand. Figure 3 illustrates that, other things being equal, the more inelastic the demand is, the larger the magnitude of the inventory and price responses to the underlying shocks, especially to the expectations shock[11]. While the larger magnitude of the price response under less elastic demand is straightforward to understand, the larger magnitude of the inventory response needs more discussion. Take the impulse response function to the temporary shock y_t^c for example. A negative temporary shock (stronger demand relative to supply) will result in an immediate increase in the spot price (P_t) and withdrawal of the inventory (n_{t+1}). Suppose the magnitude of the inventory response remains the same regardless of the price elasticity. This implies the oil availability remains the same for the next period. However, with a lower price elasticity of demand the current price (P_t) increase is larger, so is the expected spot price $(E(P_{t+1}))$. Overall the relative increase of the spot price compared to the expected future price $(P_t - E_t(P_{t+1}))$ is larger with a lower elasticity. This implies higher opportunity cost for inventory holding (see Equation 3); in other words, the inventory is too high after the assumed inventory withdrawal. Thus the inventory

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[11]Aside from γ, the three cases in Figure 3 all have the same parameters setting as listed in Table 1

(n_{t+1}) has to be drawn down more to bring the market back into equilibrium.

This "magnifying role" of the price elasticity of demand implies that when the market demand is very inelastic, the rational forward-looking behavior is more possible to result in highly volatile price movements, in a similar way Baumeister and Peersman (2013) argue that small supply or demand disturbances can generate large price responses.

To summarize, the expectations factor differs from contemporaneous factors in more than the immediate effect. The structural model is able to use the additional information of the dynamic shape and the magnitude of the responses in the identification of the expectations. In the next section, the model is brought to data and the shocks behind oil price fluctuations are estimated.

4   Estimation Results

This section presents the data and the model estimation. The estimation results include the parameter estimates, the estimated impulse response functions, the estimated underlying shocks and their contribution to the price and inventory dynamics.

4.1   Data and Estimation

4.1.1   Data

The model is estimated using monthly data from 1988 March to 2014 November. The estimation uses the real spot and futures (1-month) prices, the effective inventory and the world crude oil supply growth rate.

The estimation uses not only the spot price and inventory data, but also the 1-month futures price. The finance literature on speculation in the financialization of commodity markets highlights the spot and futures market interaction, and the information content

of the futures prices. The futures market data could provide additional information on market expectations.[12]

An overview of the data is presented in Figure 5.[13] For the prices (P_t and $E_t P_{t+1}$) the estimation uses real spot and futures (1-month) prices of WTI deflated by monthly US CPI (1982-84=100) (see Appendix A.1).

For the effective inventory n_{t+1}, the ratio of the world inventory and the world supply is used in the model solution. While the world inventory of crude oil is not available, OECD inventory is used as its proxy, which is end-of-month US commercial inventory of crude oil scaled by the ratio of OECD to US petroleum products stock, following Hamilton (2009a), Kilian and Murphy (2014) and Juvenal and Petrella (2014). The seasonality in the effective inventory data is also adjusted by including additional monthly dummies in the state equation (see Appendix A.2).

For the world crude oil supply growth rate $\log(\mu_t^s)$, the estimation uses the log first-difference of the world supply, which is available from Energy Information Administration (EIA).

4.1.2   Which Parameters are Estimated and Why γ is Arbitrarily Set

The parameters estimated are listed in Table 2 and 3. The solved linearized model allows for estimation of the parameters for the shock processes (ρ's and σ's), the parameters in the net marginal cost of inventory holding (δ and Θ' in Equation 13[14]) and the monthly dummies for the effective inventory.

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[12]Using 1-month WTI futures price for $E_t P_{t+1}$ in the model assumes that there's no risk premium in the 1-month futures price. Given the short maturity length, this assumption is not unreasonable.

[13]The log spot price and effective inventory data have been demeaned for the estimation, as the model to be estimated has all variables written in their log deviations from the steady state (See earlier section on model solution and Appendix A)

[14]Appendix A shows that the log-linearized model no longer contains Θ but only its first derivative Θ' evaluated at the steady state, which is assumed to be a constant (see discussion in 2.2). Similarly, ϕ and δ always appear together as $\phi(1 - \beta + \delta)$ and cannot be identified separately. Thus, only δ is estimated and ϕ is arbitrarily set as estimated by Pindyck (1994).

Two scalars, α in the net marginal inventory cost function, and c in the world demand for oil, are calibrated from the steady state condition using the estimated parameters and the data. This is because α and c only matter to the levels of the variables, not their deviations from the steady state. Once the model is linearized around the steady state and the variables are written in terms of their deviations from the steady state, α and c no longer appear and do not matter to the dynamics of the deviations[15]. As result, they cannot be estimated using the logged demeaned data presented in Figure 5.

Two key parameters, γ, the short-run price elasticity of demand for crude oil, and β, the monthly depreciation rate, have to be arbitrarily set as they cannot be estimated without any data on the demand. However, as discussed earlier the demand elasticity is potentially important for the estimation. Thus the range in the literature on demand elasticity estimation is used as a reference: 0.05 to 0.44 (Dahl (1993), Cooper (2003), Baumeister and Peersman (2013), Bodenstein and Guerrieri (2011), Kilian and Murphy (2014))[16] with admissible values as low as 0.01 (see Baumeister and Peersman (2013)). The literature average 0.25 is picked for γ and the results from a lower-bound 0.02 for robustness is also presented. The monthly depreciation rate is set to be 0.997.

4.2 Estimated Parameters and Impulse Response Functions

4.2.1 Estimated Parameters

Tables 2 and 3 summarize the estimation results under different demand elasticity settings[17]. In Table 2, for both cases ($\gamma = 0.25$ and 0.02) all parameter estimates are significant at 99% confidence level. In Table 3, estimates of the monthly dummies

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[15]Appendix A presents the log-linearized model and shows that it no longer contains α and c.

[16]See Hamilton (2009a) for a summary of the estimates in the literature in Table 1. Kilian and Murphy (2014) also provides a brief survey of the estimates.

[17]The model is estimated by maximum likelihood and various initial guesses of the parameters have been tried. The estimation results presented here have the highest likelihood.

indicate that effective inventory tend to be higher during colder months than warmer months (dummies for colder months tend to be negative)[18]. However, the dummies estimates are significant only for the case of $\gamma = 0.25$, though the point estimates for both cases are similar.

4.2.2 Estimated Impulse Response Functions

Figure 6 plots the impulse response functions of the price and inventory under different γ settings. All shocks are one-standard deviations, normalized to cause an increase in the real spot price of oil. Both sets of impulse response functions overall show the same direction of immediate changes and qualitatively same time paths as discussed earlier[19].

Furthermore, the estimated dynamics under $\gamma = 0.25$ shows high persistence in the persistence shock y_t^τ and the expectations shock n_t^τ, as presented in Table 2.[20] This is consistent with the high persistence in the price movement during the sample period. In the robustness check under $\gamma = 0.02$, the estimated dynamics are qualitatively similar. However, the lower-bound demand elasticity γ indeed works as a magnifier, and the estimated shocks tend to have either smaller standard deviation or lower persistence in order to reconcile with the observed price and inventory volatility, as presented in Table 2.

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[18]Similarly, Byun (2012) finds a higher utilization of inventory in refining production for warmer seasons.

[19]For example, in both cases, the persistent shock causes immediate positive changes in the spot price and negative expected changes in price ($E(P_{t+1} - P_t)$).

[20]Specifically, the price response to the expectations shock when $\gamma = 0.25$ reaches its peak after more than 60 periods.

4.3 Estimated Cumulative Effects of the Shocks

The historical decomposition results match the general understanding of the market and show that the contribution of the expectations shocks is limited. In some cases, the results even match the specific date of historical events.

Figure 7 plots the decomposed contribution of each shock on the observed real spot price and the effective inventory when $\gamma = 0.25$.[21] Overall, under the assumption of $\gamma = 0.25$, the model estimates a persisting, tight market after 2000 as indicated by the cumulative effect of the persistent shock: the persistent shock contributes to most of the price increase after 2000, except for a short period during 2008-2009 and towards the very end of the sample period (November 2014); it also contributes to the continuing withdrawal of the effective inventory, especially in 2000-2008. The model also estimates an expectation of tight market condition at the beginning of the sample period, and after January 2005: the expectation shock contributes to the price increase at the beginning of the sample period (from March 1988), and also after 2005 though to a smaller extent; it also contributes to the accumulation of the effective inventory at the beginning of the sample period and since 2004.

The results show that the price movements are mainly driven by the persistently tight market. Though the market expectations also drove up the price after 2005, quantitatively this contribution is limited compared to the overall magnitude of the price increase. The overall lack of inventory accumulation after 2000 is the result of the inventory depletion due to the persistently tight market after 1998 dominating the

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[21]It is worth noting that in this model the state variables include both the effective inventory and the exogenous shocks. As a result, to separate out the effect of a certain exogenous shock from that of the initial effective inventory and other shocks, the cumulative effect of a shock is calculated as the hypothetical price and inventory series given the Kalman-smoothed time series of the shock of interest, keeping the initial effective inventory and all other shocks as zeros. Thus, the historical decomposition of the price is sometimes negative (meaning that the price is lower than it otherwise would have been due to the shock), and that of the inventory always starts from zero in all figures. More details are provided in Appendix B.

inventory accumulation in expectation of future tight market after 2004. This suggests an overall shift of the market expectations in 2000s. The results also suggests that the expectations shock contributes more to the fluctuations in the inventory, rather than to the fluctuations in the price. Kilian and Murphy (2014) have a similar observation of their speculative demand shock.

The robustness check results are similar. Figure 8 plot the same when $\gamma = 0.02$. Under the extreme assumption of demand elasticity ($\gamma = 0.02$), the estimated cumulative effect of the persistent shock is similar as in the case of $\gamma = 0.25$. The model also estimates similar pattern for the cumulative effect of the expectations shock: the expectations shock contributes to a price spike in August 1990 (the outbreak of the Gulf War); it also contributes to the accumulation of the effective inventory in October 1990, and after 2004 except for the period from July 2008 to March 2009 (the oil price peaked in June 2008).

Again, the results show that the price movements are mainly driven by the persistently tight market. However, the estimation does attribute relatively more of the price movements to the expectations shock compared to when $\gamma = 0.25$, due to the magnifying role of the price elasticity of demand.

To illustrate and compare their relative contribution, Figure 9 rearranges the plottings and compares the historical decomposition under different elasticities side by side. The comparison confirms that the overall patterns of the decomposed cumulative effects are similar, and difference in the magnitude is small. Overall, in both cases, the persistent shock is the largest contributor for the price dynamics, followed by the temporary shock, and the expectation shock; the temporary shock is the largest contributor to the effective inventory fluctuations.

The variance decomposition results in Table 4, which reflect the average contribution of each type of shocks, show that overall the expectations shock is estimated to

contribute to less than 1% of the price movements. When the price elasticity of demand is assumed lower, the estimation indeed attributes relatively more importance on the expectations shock. But contributing by 3.5%, the expectations shock still cannot be the main driver.

The estimated contribution of the expectations shock is lower compared to the literature, where Kilian and Murphy (2014) estimate 9% of long-run price variance to be due to speculative demand shocks and Juvenal and Petrella (2014) estimate 10% - 30%. This is because the expectations shock in this paper is defined such that it does not cause any immediate changes in the relative supply, which is different from the two papers mentioned above.

5 Conclusion

This paper models market expectations explicitly in a structural model where the equilibrium prices and inventory are endogenously determined. The expectation of future market condition is explicitly modelled as a shock that affects the relatively supply with a delay, in order to capture the forward-looking component in the price formation. Bringing the model to data, it is possible to analyze the contribution of expectations in the oil price dynamics.

This model contributes to the discussion on the role of speculation in commodity price dynamics by bridging the classic theory of storage and the macroeconomic literature on the news shock in order to capture market expectations. In the empirical application, this paper also attempts to incorporate insights of the finance literature on speculation in the financialization of commodity markets, which approaches speculation from the perspective of the spot and futures markets interaction.

The model simulation reveals rich dynamics of the way expectations affect the price

and inventory dynamics, which enriches the previous literature. The model simulation also shows that the price elasticity of demand plays a key role.

Under reasonable assumption of the price elasticity of demand (-0.25), the oil price movements have been mostly driven by a persisting, constrained supply relative to demand especially since 2000s. In addition, the short-run movements in the effective inventory are mostly contributed by the temporary shock, while the long-run trend in the relative inventory is driven by the persistent shock and the expectations shock together. The robustness check assuming an extremely low elasticity (-0.02) also has similar results.

While the current version of the model finds little evidence for the expectations driving up the price in the 2000s, this could have to do with how expectation is modelled. The expectations shock is a shock to the relative supply with a lag, and thus captures expectations of the future level of relative supply. However, the speculative incentives also include increased uncertainty about future market condition, which can be modelled as a mean-preserving volatility increase of the relative supply. This would affect prices and inventory decision without changing future relative supply, which cannot be captured by the current expectations shock. As Kilian and Murphy (2014) point out, "news about the level of future oil supplies and the level of future demand for crude oil are but one example of shocks to expectations in the global market for crude oil." Such mean-preserving volatility-increasing expectations shock can be explored in the future work.

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A Solving the Model

To solve the detrended model in Section 2.4, first, its steady state is found and the model is log-linearized around the steady state, then the log-linearized linear system is solved using Blanchard and Kahn (1980) and the model solution is written in a state-space form.

First, the steady state of the model in Section 2.4 is written as follows (the steady state values are in bold; for example $n_t = n_{t+1} = \boldsymbol{n}$ in steady state):

$$\boldsymbol{P} = c[(\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - n) * \boldsymbol{q^s}]^{-\frac{1}{\gamma}} \tag{14}$$

$$1 = \beta - [\alpha(\frac{\boldsymbol{n}/\boldsymbol{\mu^s}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}})^{-\phi} + \delta] \tag{15}$$

$$\log \boldsymbol{\mu^s} = \bar{\mu} \tag{16}$$

$$\log \boldsymbol{q^s} = 0 \tag{17}$$

$$\boldsymbol{y^\tau} = 0 \tag{18}$$

$$\boldsymbol{y^c} = 0 \tag{19}$$

$$\boldsymbol{n^\tau} = 0 \tag{20}$$

Then the model in Section 2.4 is log-linearized around the steady state.

Define $\hat{P}_t = (P_t - \boldsymbol{P})/\boldsymbol{P}$, $\hat{n}_t = (n_t - \boldsymbol{n})/\boldsymbol{n}$, $\hat{\mu}_t^s = (\mu_t^s - \boldsymbol{\mu^s})/\boldsymbol{\mu^s}$, $\hat{q}_t^s = (q_t^s - \boldsymbol{q^s})/\boldsymbol{q^s}$ for all t, the original model in Section 2.4 can be written as terms of the deviation from the steady state:

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$$\hat{P}_t = -\frac{1}{\gamma}[pn_0\hat{n}_t - pn_1\hat{n}_{t+1} - pu\hat{\mu}_t^s + py\hat{q}_t^s] \tag{21}$$

where

$$pn_0 = \frac{\boldsymbol{n}/\boldsymbol{\mu^s}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} \tag{22}$$

$$pn_1 = \frac{\boldsymbol{n}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} \tag{23}$$

$$pu = \frac{\boldsymbol{n}/\boldsymbol{\mu^s}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} \tag{24}$$

$$py = 1 \tag{25}$$

$$\hat{P}_t = \beta E_t[\hat{P}_{t+1}] - \frac{\boldsymbol{MIC}}{\boldsymbol{P}} E_t[\hat{MIC}_{t+1}] \tag{26}$$

where

$$\hat{MIC}_{t+1} = \hat{P}_t + micn_0\hat{n}_t + micn_1\hat{n}_{t+1} + micn_2\hat{n}_{t+2} + micu_0\hat{\mu}_t^s + micu_1\hat{\mu}_{t+1}^s \tag{27}$$

$$micn_0 = -\frac{1}{\beta - 1} * \Theta' * \boldsymbol{\mu^s} \tag{28}$$

$$micn_1 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{1 - \boldsymbol{n}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} + (1 + \beta) * \Theta' * \boldsymbol{\mu^s}] \tag{29}$$

$$micn_2 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{\boldsymbol{n}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} - \beta * \Theta' * \boldsymbol{\mu^s}] \tag{30}$$

$$micu_0 = \frac{1}{\beta - 1} * \Theta' * \boldsymbol{\mu^s} \tag{31}$$

$$micu_1 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{\boldsymbol{n} - 1}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} - \beta * \Theta' * \boldsymbol{\mu^s}] \tag{32}$$

Following Blanchard and Kahn (1980), the log-linearized model's variables are

grouped as state variables X_t, costate variables Y_t and exogenous shock variables e_t, where $X_t' = \begin{bmatrix} \hat{n}_t & \hat{n}_{t+1} \end{bmatrix}$, $Y_t = \begin{bmatrix} \hat{P}_t \end{bmatrix}$, $e_t' = \begin{bmatrix} \hat{\mu}_t^s & y_t^\tau & y_t^c & n_t^\tau \end{bmatrix}$. The above model can be solved for the state-space form (or more specifically, to solve for F, Z, U, H and R in the state-space form below from Equation (21) - (32)).

The resulting state-space model is in the format below:

State equation:

$$
\begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} = F \begin{bmatrix} \hat{n}_{t-1} \\ e_{t-1} \end{bmatrix} + Z * v_t \qquad v_t \sim N(0, U) \tag{33}
$$

where $v_t' = \begin{bmatrix} \epsilon_t^{\mu^s} & \epsilon_t^{y_\tau} & \epsilon_t^{y_c} & \epsilon_t^{n_\tau} \end{bmatrix}$, $Z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $U = \begin{bmatrix} \sigma_{\mu^s}^2 & 0 & 0 & 0 \\ 0 & \sigma_{y_\tau}^2 & 0 & 0 \\ 0 & 0 & \sigma_{y_c}^2 & 0 \\ 0 & 0 & 0 & \sigma_{n_\tau}^2 \end{bmatrix}$.

Observation equation:

$$
\hat{P}_t = H \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + u_{1t} \qquad u_t \sim N(0, R_1) \tag{34}
$$

where u_{1t} is the measurement error for the spot price, and its variance is a small positive number (in the estimation it is set to be $1/100000$).

A.1 Additional Observables

In addition to the spot market, crude oil futures contracts are also actively traded. If 1-month futures price approximates of the expected 1-month ahead spot price, the futures price can serve as another observed variable.

The state space model implies the following for the 1-month ahead expected price:

$$E_t \hat{P}_{t+1} = H \begin{bmatrix} E_t \hat{n}_{t+1} \\ E_t e_{t+1} \end{bmatrix} \tag{35}$$

$$= H * F * \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} \tag{36}$$

This gives rise to the second observation equation:

$$\hat{F}_{t,1} = H * F * \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + u_{2t} \qquad u_t \sim N(0, R_2) \tag{37}$$

where $F_{t,1}$ is the 1-month futures price quoted at t and u_{2t} is the measurement error for the futures price, and its variance is a small positive number (in the estimation it is set to be $1/100000$).

A.2   Observable State Variables

One advantage of the model is that two of the state variables are actually observed. Both the effective inventory $\hat{n}_{t+1}$ and the world supply growth rate $\hat{\mu}_t^s$ are available. This provides two additional observation equations in the state-space form:

$$\begin{bmatrix} \hat{n}_t \\ \hat{\mu}_t^s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + \begin{bmatrix} \epsilon_t^{\hat{n}} \\ 0 \end{bmatrix} \qquad \epsilon_t^n \sim N(0, \sigma_{\hat{n}}^2) \tag{38}$$

where ϵ_t^n is the measurement error for the effective inventory. This allows for correcting possible data inaccuracy due to using the OECD effective inventory as the proxy of world inventory. On the other hand, the dynamics of world supply growth rate $\hat{\mu}_t^s$ is

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already modelled in the state equation (see Equation 33) and already contains a shock $\epsilon_t^{\mu^s}$, thus the observation equation does not include any error term for $\hat{\mu}_t^s$.

In order to remove the seasonality in the inventory data, 11 monthly dummies are included in the inventory observation equation, so that in the estimation:

$$\hat{n}_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + d_i + \epsilon_t^{\hat{n}} \quad \epsilon_t^n \sim N(0, \sigma_{\hat{n}}^2) \tag{39}$$

where d_i is the dummy variable for month i, with March excluded.

A.3 Equations for the Estimation

To summarize, the equations in the estimation are Equations 33 34 37 39 and the second row (for $\hat{\mu}_t^s$) of Equation 38.

B Estimation of the State Space Model

Given a starting set of parameters, with the state equation 33, the observation equations 34 37 39 and the second row (for $\hat{\mu}_t^s$) of equation 38, and the observed data, the Kalman filter is used to produce the estimates of the state variables, as well as the joint likelihood under this set of parameter. The maximum likelihood estimation of the model involves finding the parameters to maximize the joint likelihood. Once the parameters are estimated, the estimates of the state variables are also produced, and smoothed by Kalman smoother. The state variables and the decomposition results discussed in the paper are all based on smoothed state variables.

For the results discussion, the smoothed state variables are not plotted. Rather the historical decomposition and variance decomposition are provided for better illustration. The figures of the state variables can be provided on request.

To compute the historical decomposition of the price and inventory, aside from the shock of interest, all other shocks are set to be zeros over the whole sample period. The effective inventory in the first period is also set to be zero. The hypothetical price and inventory over time is calculated iteratively from the time path of the shock of interest, using the estimated state space model. Thus the historical decomposition of the price is sometimes negative (meaning that the price is lower than it otherwise would have been due to the shock), and that of the inventory always starts from zero in all figures.

Table 1: **Model Parameterization**

Parameters	Value	Description
β	0.997	monthly depreciation rate
γ	0.25	price elasticity of demand
ϕ	1.42	parameter in MIC
Θ'	0.2	marginal cost of inventory change
δ	0.89	marginal physical storage cost
ρ^τ	0.9	AR coef of persistent shock
ρ^c	0.1	AR coef of temporary shock
ρ^{n_τ}	0.5	AR coef of expectation shock
σ_{y_τ}	1	s.d. of persistent shock
σ_{y_c}	1	s.d. of temporary shock
σ_{n_τ}	1	s.d. of expectation shock
σ_{μ^s}	1	s.d. of growth rate shock
$\sigma_{\hat{n}}$	1	s.d. of inventory measurement error[a]

[a]In the observation equation, although the observed effective inventory is mapped 1 to 1 directly from the state variable effective inventory, measurement errors in the observed values is allowed.

Parameters	$\gamma = 0.25$		$\gamma = 0.02$		Description
	Point Estimate	(Standard Error)	Point Estimate	(Standard Error)	
log likelihood	4628		4635		
β (set)	0.997		0.997		monthly depreciation rate
γ (set)	0.25		0.02		price elasticity of demand for crude oil
ϕ (set)	1.42		1.42		parameter in net marginal convenience yield
Θ'	0.0151***	(0.0004)	0.0018***	(0.0002)	
δ	0.0025***	(0.0001)	0.0021***	(0.0001)	marginal physical storage cost
ρ^τ	0.9993***	(0.0000)	0.9998***	(0.0000)	AR coefficient of persistent shock
ρ^c	0.0451***	(0.0035)	0.0279***	(0.0011)	AR coefficient of temporary shock
ρ^{n_τ}	0.9991***	(0.0000)	0.0000***	(0.0000)	AR coefficient of expectation shock
σ_{y_τ}	0.0197***	(0.0001)	0.0010***	(0.0002)	s.d. of persistent shock
σ_{y_c}	0.0092***	(0.0003)	0.0088***	(0.0015)	s.d. of temporary shock
σ_{n_τ}	0.0000***	(0.0000)	0.0003***	(0.0000)	s.d. of expectation shock
σ_{μ^s} (set)	0.0105		0.0105		s.d. of growth rate shock
$\sigma_{\hat{n}}$	0.0000***	(0.0000)	0.0000***	(0.0000)	s.d. of inventory measurement error[a]

Note: (i) Standard errors of the estimates are simulated and reported in parentheses; (ii) *, ** and ***denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

Table 3: **Estimated Model for Crude Oil Market - continued**

Parameters	$\gamma = 0.25$		$\gamma = 0.02$		Description
	Point Estimate	(Standard Error)	Point Estimate	(Standard Error)	
log likelihood	4628		4635		
Jan.	-0.0377^{***}	(0.0052)	-0.0383	(0.0329)	monthly seasonality dummy
Feb.	-0.0105^{***}	(0.0036)	-0.0109	(0.1042)	monthly seasonality dummy
Mar.(set)	0		0		monthly seasonality dummy
Apr.	0.0300^{***}	(0.0037)	0.0305	(0.0308)	monthly seasonality dummy
May.	0.0419^{***}	(0.0050)	0.0429	(0.0307)	monthly seasonality dummy
Jun.	0.0337^{***}	(0.0060)	0.0348	(0.0309)	monthly seasonality dummy
Jul.	0.0112^{*}	(0.0063)	0.0115	(0.0312)	monthly seasonality dummy
Aug.	-0.0041	(0.0066)	-0.0040	(0.0646)	monthly seasonality dummy
Sep.	-0.0129^{*}	(0.0067)	-0.0132	(0.0440)	monthly seasonality dummy
Oct.	-0.0333^{***}	(0.0064)	-0.0339	(0.0317)	monthly seasonality dummy
Nov.	-0.0068	(0.0063)	-0.0073	(0.0353)	monthly seasonality dummy
Dec.	-0.0121^{**}	(0.0057)	-0.0130	(0.0480)	monthly seasonality dummy

Note: (i) Simulated standard errors of the estimates are in parentheses (20000 simulations); (ii) *, **
and ***denote that the point estimate is significant at the 90%, 95% and 99% confidence levels,
respectively.

Table 4: **The Variance Decomposition k-month Ahead under Different γ's**

Forecast error in	Innovation in	γ	$k=1$	$k=3$	$k=6$	$k=12$	$k=24$
P_t	y^τ	$\gamma = 0.25$	0.9967	0.9974	0.9976	0.9978	0.9975
		$\gamma = 0.02$	0.9501	0.9515	0.9526	0.9546	0.9573
	y^c	$\gamma = 0.25$	0.0013	0.0007	0.0004	0.0002	0.0000
		$\gamma = 0.02$	0.0109	0.0096	0.0086	0.0068	0.0043
	n^τ	$\gamma = 0.25$	0.0002	0.0002	0.0002	0.0003	0.0008
		$\gamma = 0.02$	0.0350	0.0351	0.0352	0.0354	0.0357
n_{t+1}	y^τ	$\gamma = 0.25$	0.0016	0.0073	0.0261	0.1042	0.2686
		$\gamma = 0.02$	0.0000	0.0000	0.0001	0.0004	0.0017
	y^c	$\gamma = 0.25$	0.8223	0.8083	0.7600	0.5606	0.1473
		$\gamma = 0.02$	0.8123	0.8126	0.8126	0.8124	0.8115
	n^τ	$\gamma = 0.25$	0.0034	0.0155	0.0551	0.2181	0.5533
		$\gamma = 0.02$	0.0009	0.0008	0.0008	0.0007	0.0005

Note: (i) P_t: the spot price in period t; n_{t+1}: the effective inventory determined in period t for the beginning of period $t+1$; (iI) y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock.
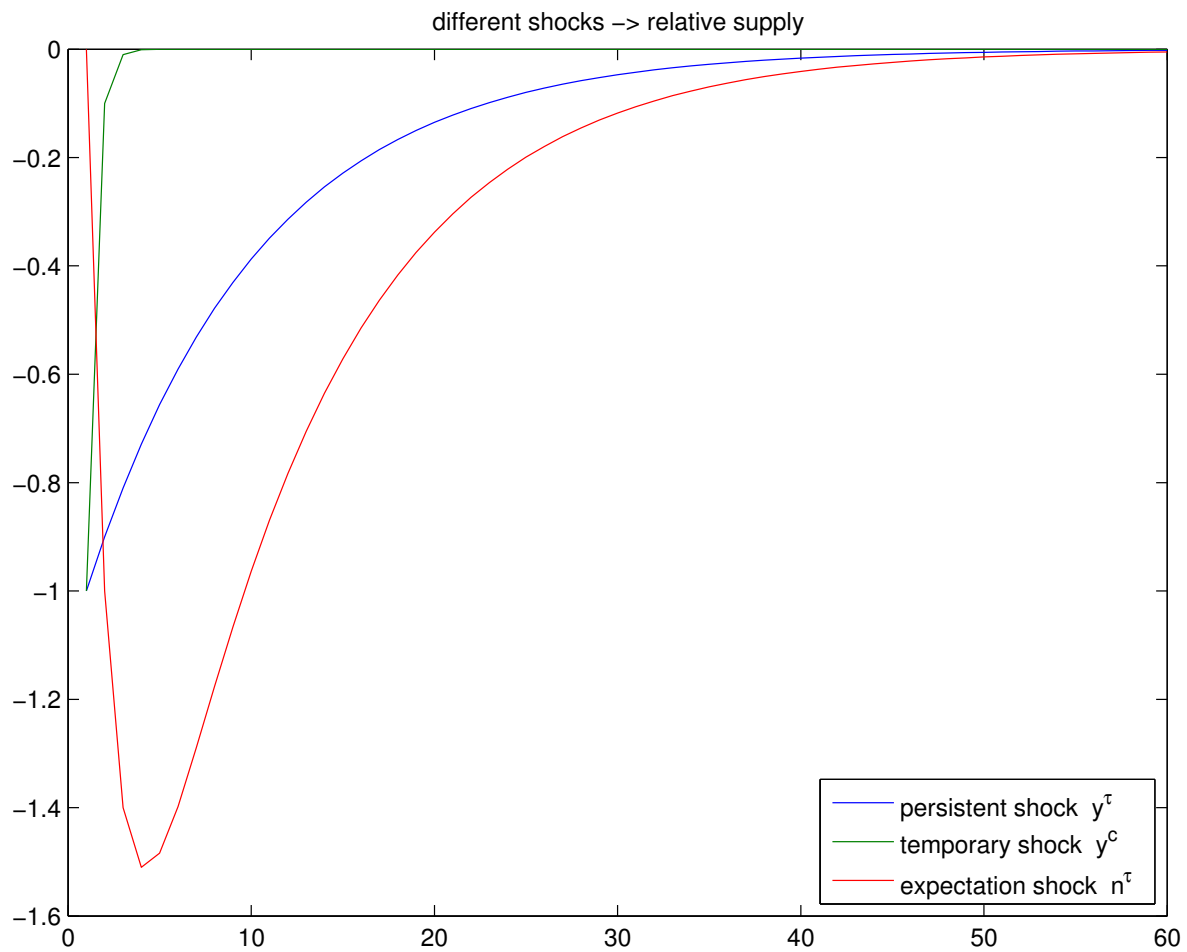
Figure 1: Effect of the Shocks on Relative Supply under Arbitrary Parameterization

Note: All shocks have been normalized to cause a decrease in the relative supply.
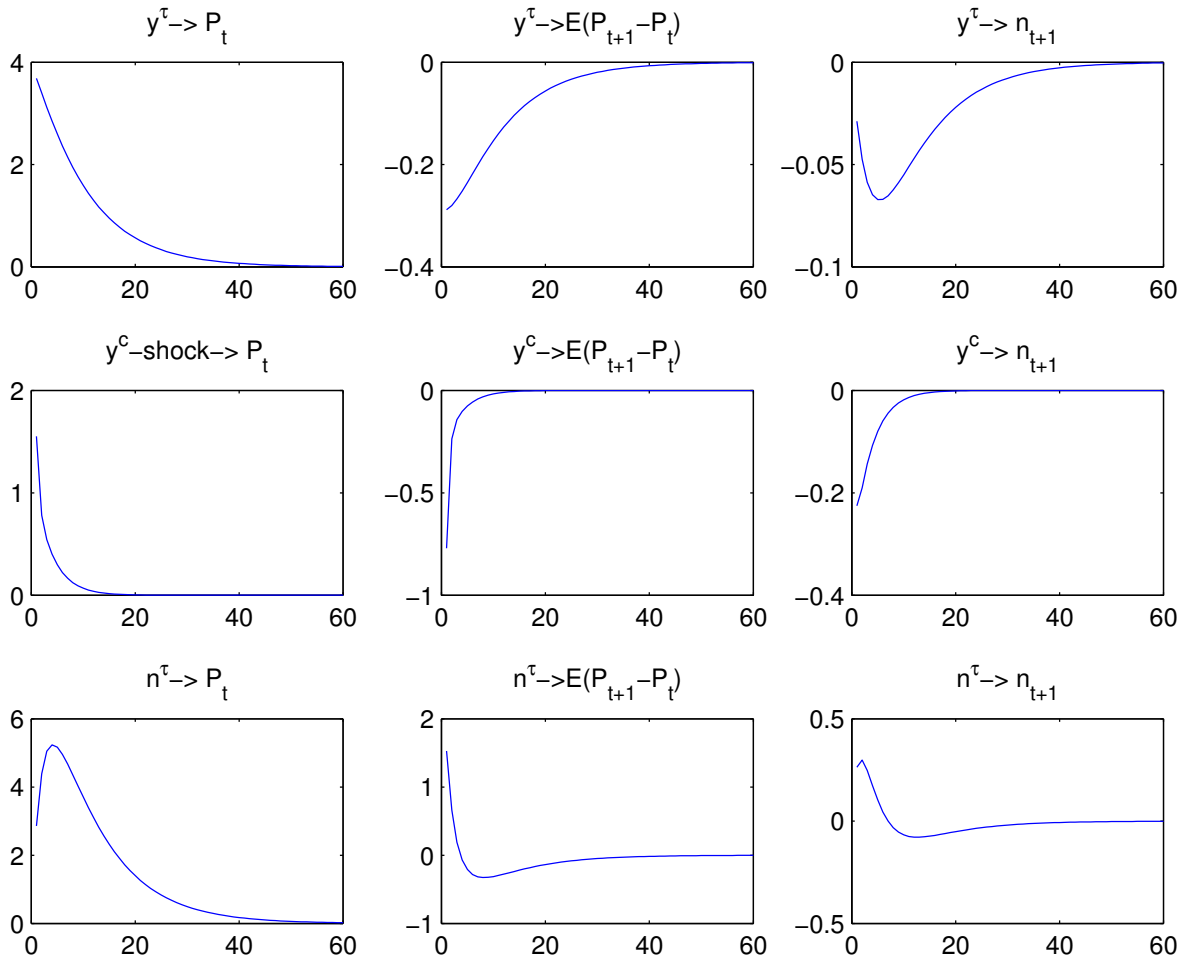
Figure 2: Impulse Response Functions under Arbitrary Parameterization

Note: 1. y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock; 2. All shocks have been normalized to cause an increase in the real spot price of oil.
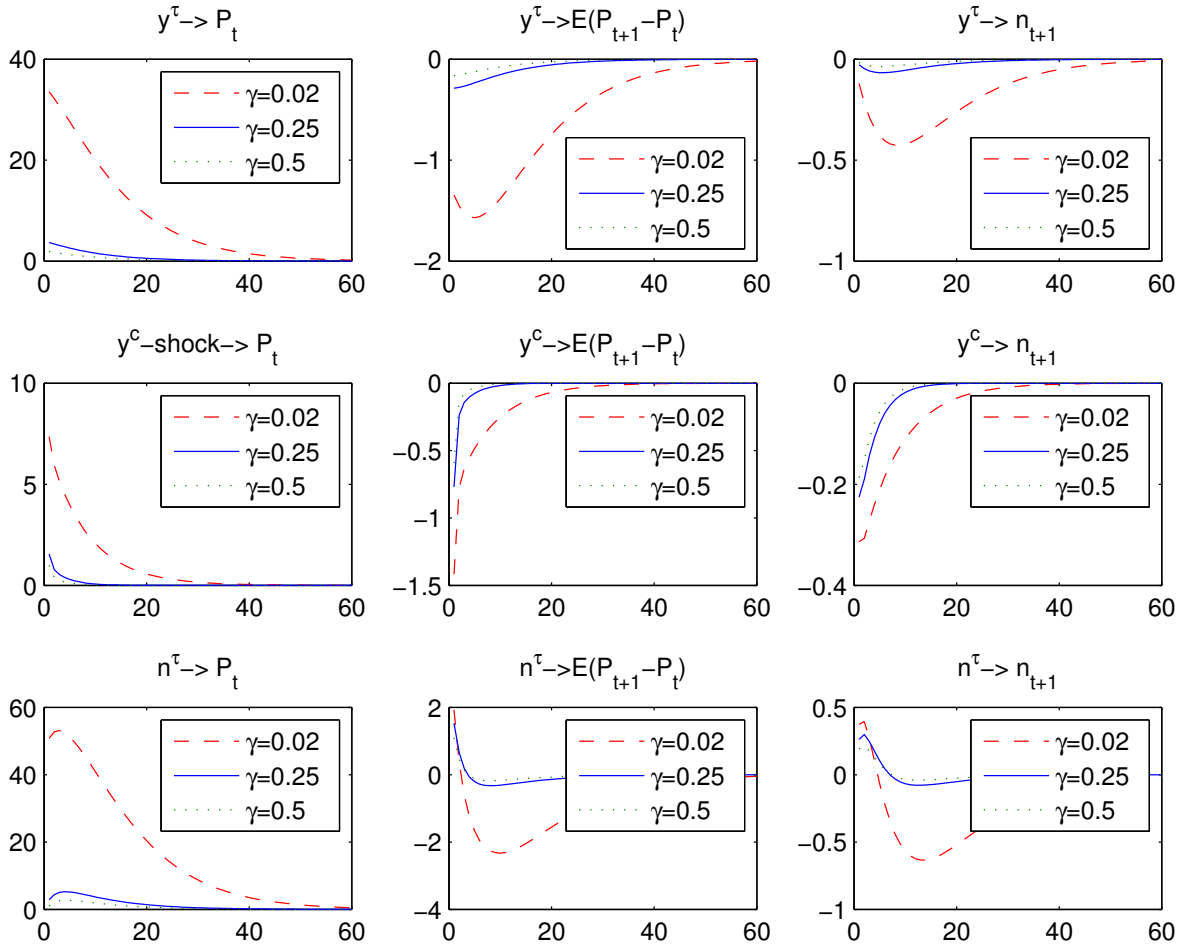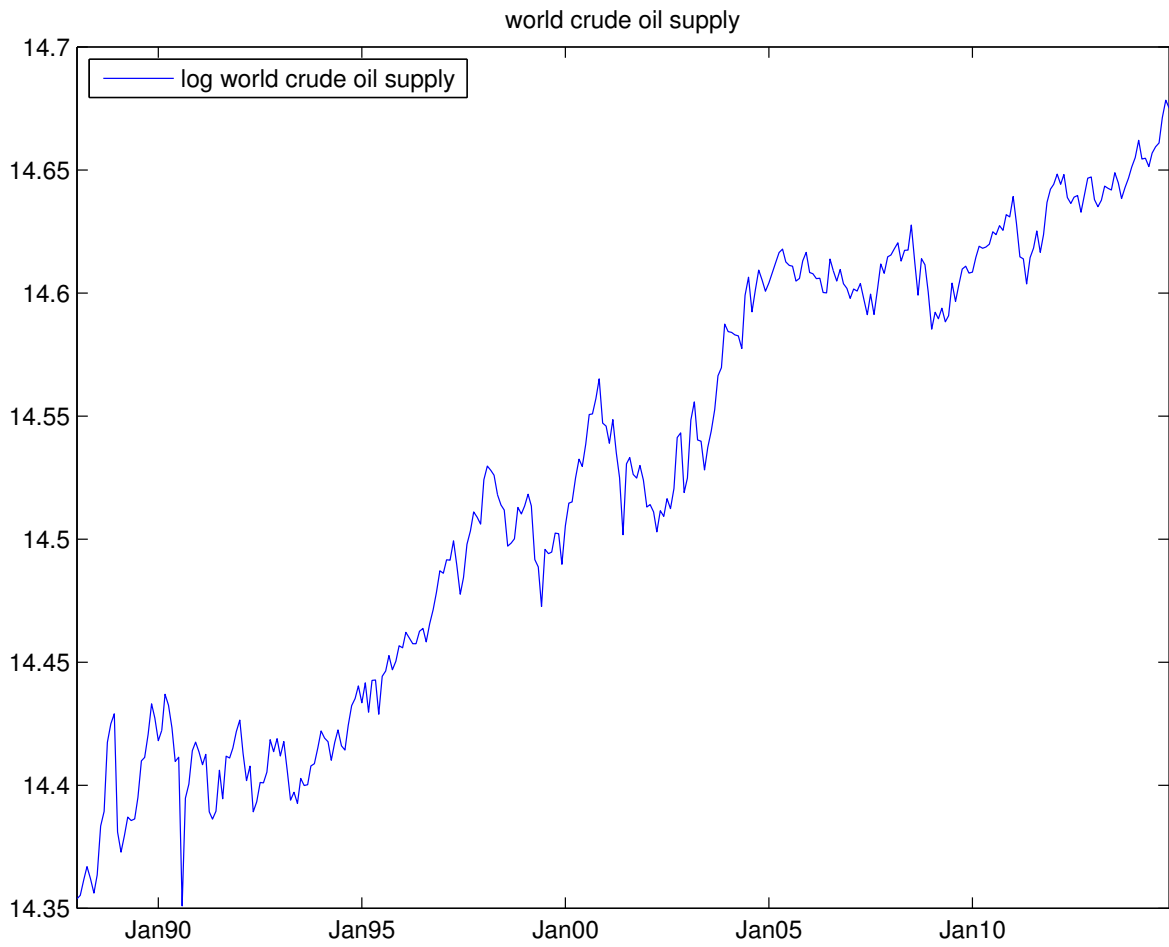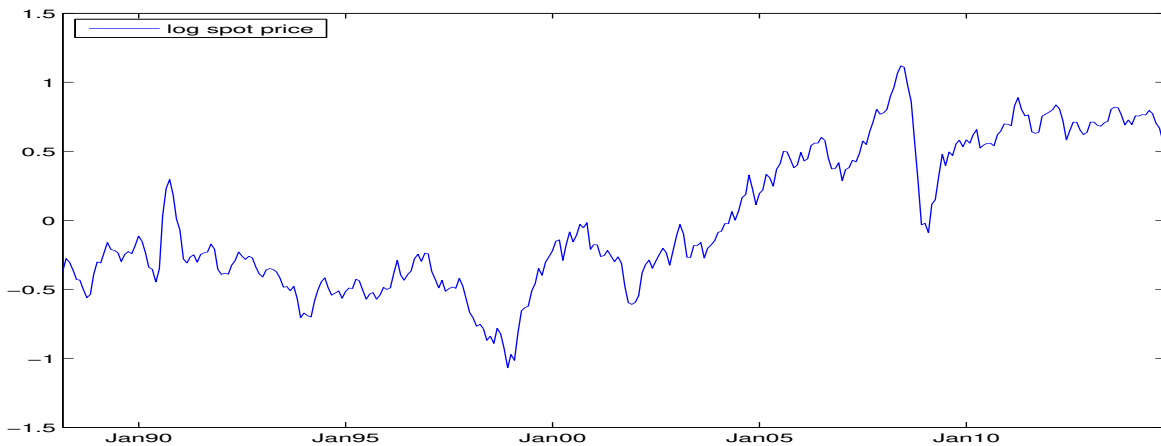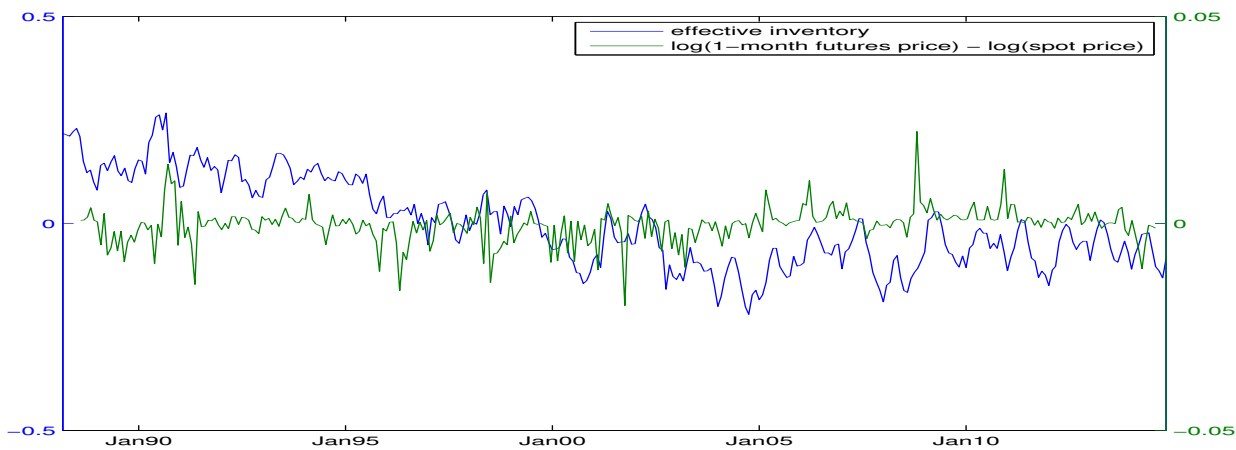
Figure 3: Impulse Response Functions under Arbitrary Parameterization with different γ's

Note: 1. y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock; 2. All shocks have been normalized to cause an increase in the real spot price of oil.

Figure 4: World Supply of Crude Oil

Source: Author's calculation. Energy Information Administration (EIA).

(a) WTI Prices

(b) Effective Inventory
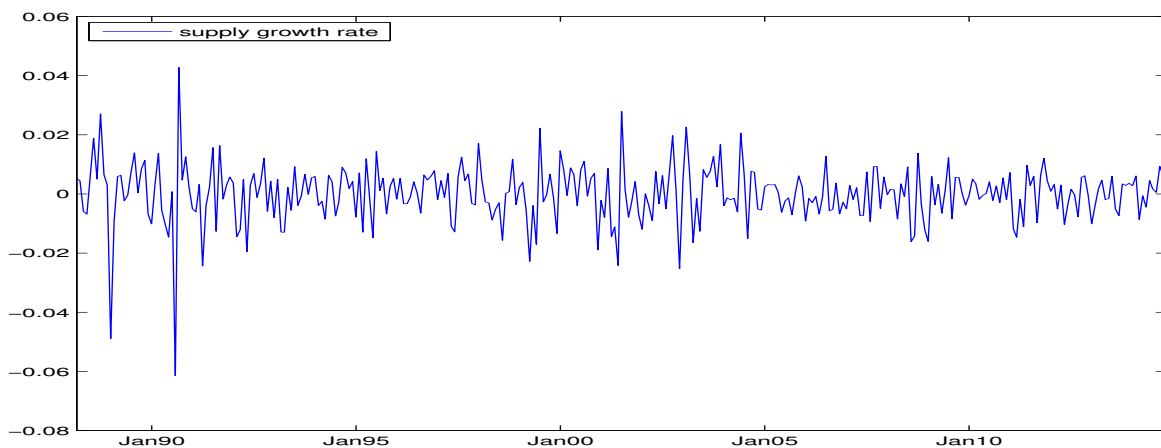
(c) Supply Growth Rate

Figure 5: Data Overview

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Source: Author's calculation. Energy Information Administration (EIA).
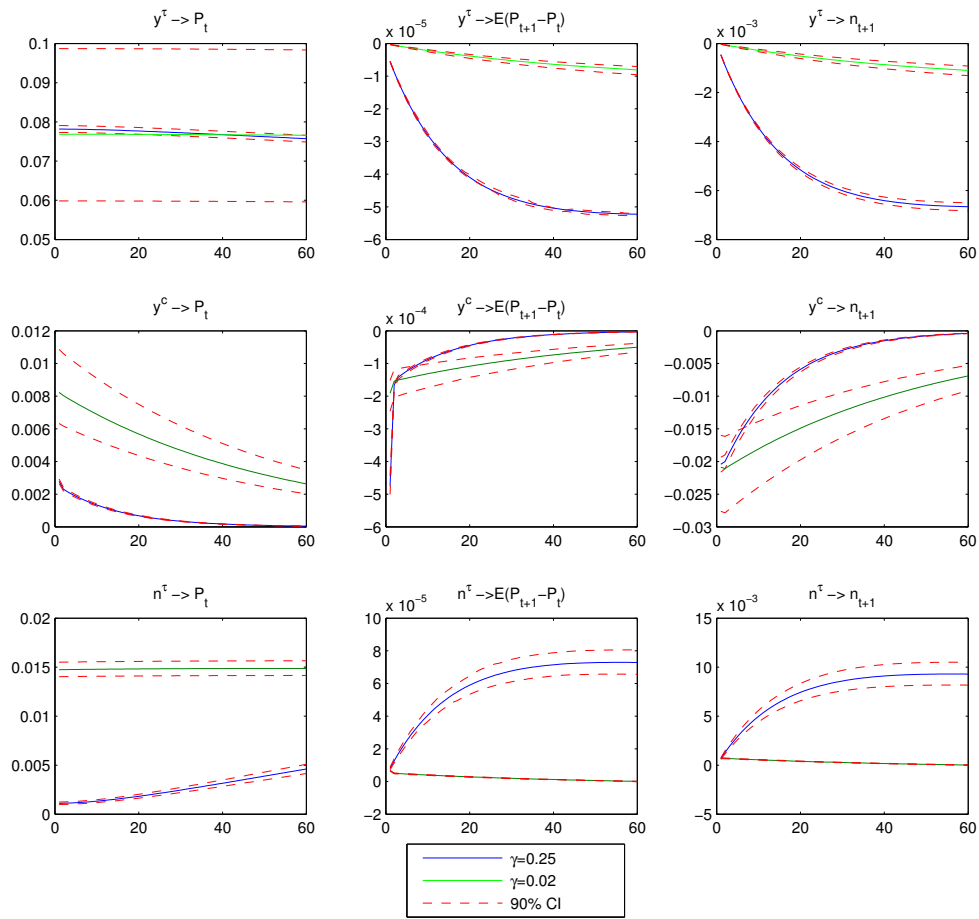
Figure 6: Estimated Impulse Response Functions

Note: 1. y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock; 2. All shocks have been normalized to cause an increase in the real spot price of oil.
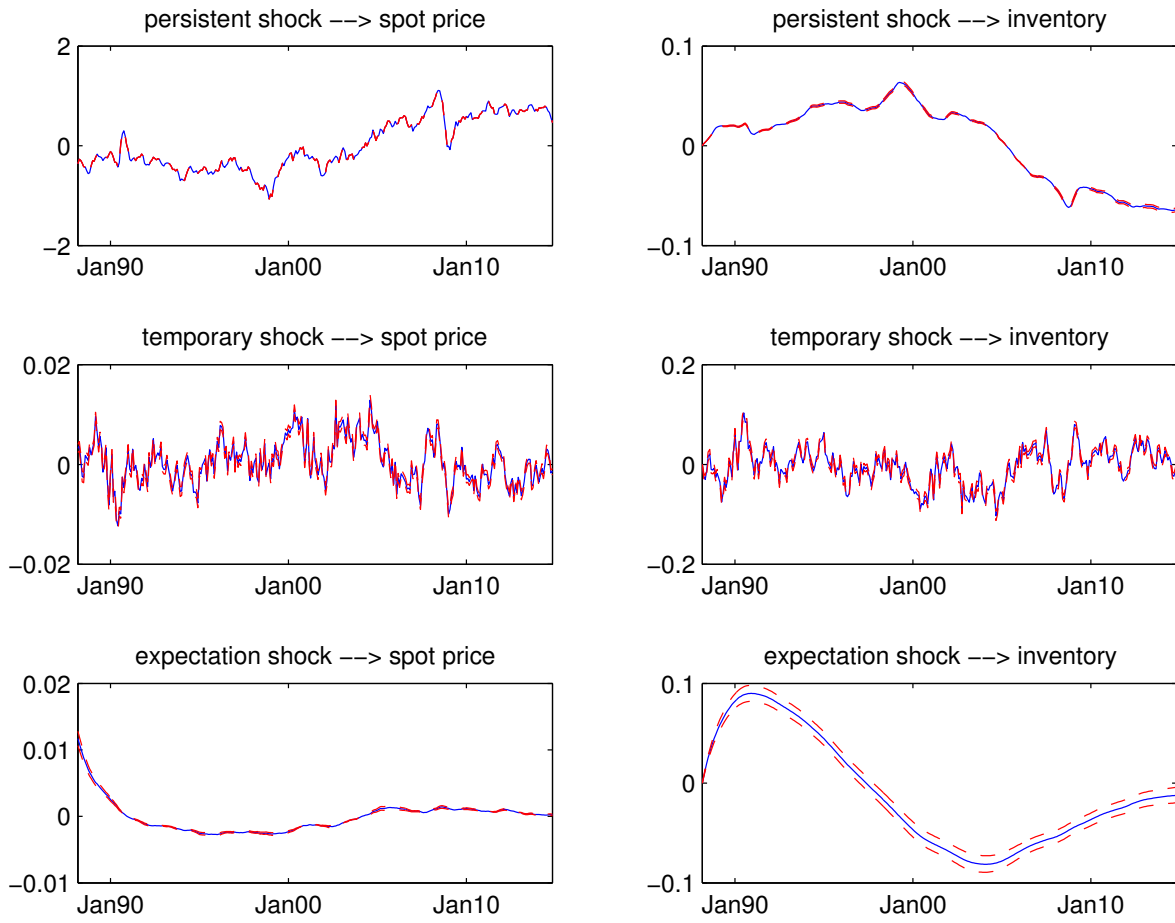
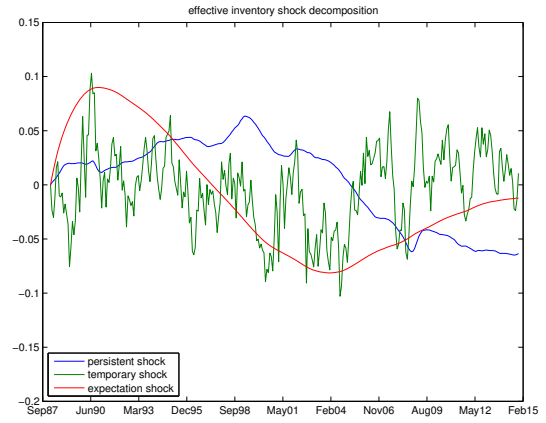Figure 7: Cumulative Effect of Shocks on the Prices and Effective Inventory with 90% CI: $\gamma = 0.25$
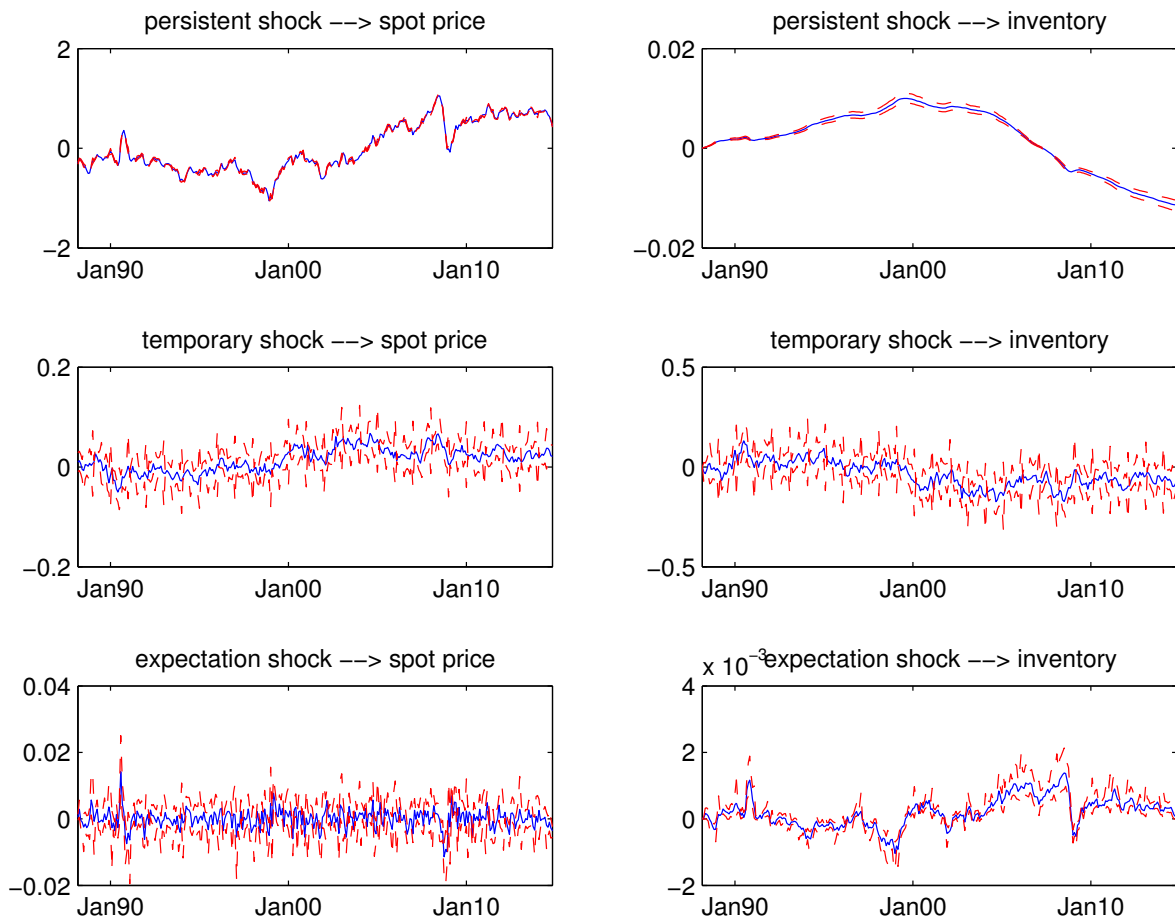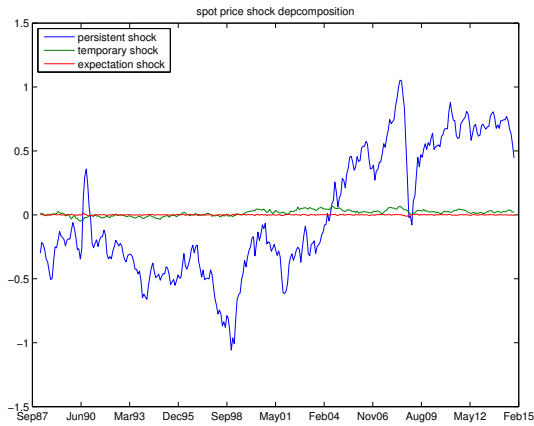
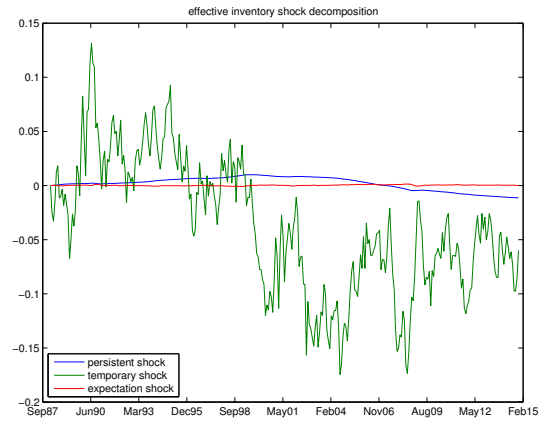Figure 8: Cumulative Effect of Shocks on the Prices and Effective Inventory with 90% CI: $\gamma = 0.02$
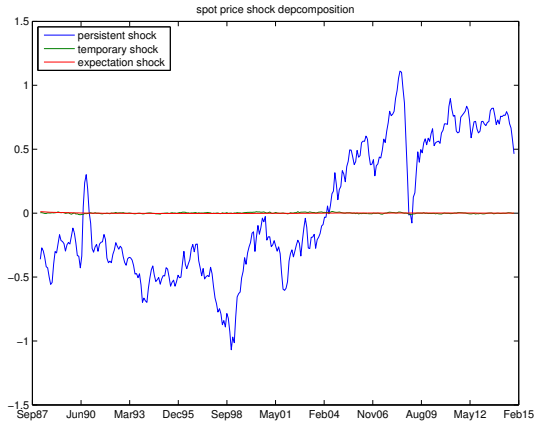
(a) $\gamma = 0.25$: Spot Price Decomposition

(b) $\gamma = 0.25$: Effective Inventory Decomposition

(c) $\gamma = 0.02$: Spot Price Decomposition

(d) $\gamma = 0.02$: Effective Inventory Decomposition

Figure 9: Cumulative Effect of Shocks to Price and Inventory

Note: For illustration purpose, the CI's from Figure 7 and Figure 8 are not included in the rearranged plottings.

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