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Satisficing and Maximizing Consumers in a Monopolistic Screening Model

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Abstract

We study a simple model in which a monopolist supplies a multi-attribute good and does not know whether the consumer is an expected-utility maximizer or a boundedly rational type that follows the satisficing heuristic proposed by Herbert Simon. We find that unless the probability of the consumer being fully rational is sufficiently high, the fact that the boundedly rational consumer never exchanges satisfactory with unsatisfactory alternatives implies that she never ends up with an alternative strictly better than her aspiration levels.

Keywords: Bounded Rationality, Monopoly, Satisficing, Welfare.
JEL classification: D6, D8, L1, L2.

1. Introduction

Ellison (2006) identifies three distinct traditions in the literature on bounded rationality and industrial organization (IO). The first one is called *rule-of-thumb* approach. This tradition, rather than characterizing equilibrium behavior, assumes that economic agents behave in some simple way. The second one is called *explicit bounded rationality* approach, assumes that cognition is costly, and derives second-best behaviors, given the costs. The third one models economic agents as individuals subject to biases typically detected in experimental economics and psychology and examines what happens in a market in which consumers or firms exhibit such a bias. Our paper belongs to the third category.

Standard models of choice assume that given a choice problem the decision-maker chooses the alternative that yields the highest utility. Experiments have challenged this paradigm by showing that often subjects' behavior is consistent with simple choice procedures (heuristics) that require less cognitive effort than standard maximization. An example is the satisficing heuristic proposed by Herbert Simon (1955), according to which the decision-maker has in mind an aspiration level and judges satisfactory all those alternatives that are at least as good as the aspiration level and unsatisfactory those that are not. The decision-maker stops searching as soon as she identifies the first satisfactory alternative and chooses it. If there is none, she might either postpone the decision or choose the best (or last) discovered alternative.

Imagine that a consumer, who wants to purchase a new car, is interested in only two attributes: price and level of emissions. Assume that she prefers to spend as little as possible and to own a car that does not pollute that much. Whether she maximizes or follows the satisficing heuristic has an effect on her choice behavior. Consider an alternative $\alpha = (p_\alpha, q_\alpha)$, where p stands for price and q for quality in terms of level of emissions.[1] If the consumer maximizes then no matter how much the price level p is increased relative to p_α within the limits imposed by the domain, one can find another alternative $\beta = (p_\beta, q_\beta)$ whose price is $p_\beta = p$ that is revealed to be indifferent to α for a sufficiently low level of emissions. On the other hand, assume that the consumer follows the satisficing heuristic. That is, she has in mind an aspiration level for both price and level of emissions. Assume that she judges α to be satisfactory. Note that if the price p is increased relative to p_α besides a certain level, then no matter how much the level of emissions is reduced the consumer will never choose $\beta = (p, q_\beta)$ over α. The reason is that if the price p exceeds the consumer's aspiration level on price, then β is unsatisfactory independently of its level of emissions and therefore strictly worse than α.

The satisficing heuristic is supported by experimental evidence. Caplin et al. (2011), for instance, propose an experiment in which they analyze the source of choice errors by recording intermediate choices and find that subjects behavior is consistent with

[1] We interpret $q_\alpha < q_\beta$ if and only if α's level of emissions is greater than β's.

a reservation-based model of sequential search. Reutskaja et al. (2011), on the other hand, use eye-tracking to investigate consumer search dynamics in a context characterized by time pressure. They show that subjects tend to choose the optimal alternative among those discovered and that search behavior is compatible with an hybrid of the optimal search and the satisficing model. The satisficing heuristic has also attracted theorists' attention. Rubinstein and Salant (2006), for instance, propose a model in which the decision-maker examines alternatives one by one. Caplin and Dean (2011), on the other hand, assume that not only final but also intermediate choices are taken into account to infer the decision-maker's aspiration level and preferences.[2]

Since as highlighted above whether the decision-maker maximizes or behaves according to the satisficing heuristic has an effect on her choice behavior, we think that it is interesting to analyze what are the normative and behavioral implications of consumers begin maximizing and satisficing in a market enivornment. We address this research question by proposing a very simple monopolistic screening model in which a monopolist supplies a two-attribute good and does not know whether the consumer is an expected-utility maximizer or a satisficing type.

We find that unless the probability of the consumer being fully rational is sufficiently high, the monopolist exploits the fact that the boundedly rational consumer is unwilling to make trade-offs between satisfactory and unsatisfactory alternatives by supplying to the satisficing type an alternative whose attributes are never better than her aspiration levels. The fact that the boundedly rational consumer sometimes avoids trade-offs (and the expected utility-maximizer does not) allows the monopolist to supply the 'minimum' to the boundedly rational consumer and, by properly combining the attributes, an unsatisfactory alternative that yields the reservation utility to the other type. We argue that one way to prevent this phenomenon independently of the distribution of types is to promote policies aimed at discouraging the fully rational consumer from considering 'extreme' alternatives.

The literature on bounded rationality and IO has been growing over the last few years (Ellison, 2006; Spiegler, 2011). The closest study to our work is Spiegler (2006),

[2] Other studies are Tyson (2008), Horan (2010), Papi (2012).

who proposes a market model in which profit-maximizing firms compete in a multidimensional pricing framework by defining boundedly rational consumers as individuals that have limited capacity of understanding complex objects. In that model each firm to an increase in competition responds by putting in practice a confounding strategy rather than a strategy of more competitive prices. Unlike Spiegler (2006), we consider a market in which only one firm is active and consumers are either satisficing or maximizing types. Studying the welfare implications of having fully and boundedly rational agents have recently attracted attention also outside IO. For example, Abdulkadiroglu et al. (2011) examine the welfare of naive and sophisticated players in school choice.[3]

The paper is organized as follows. Section 2 introduces the formal model; Section 3 contains the formal analysis; Section 4 discusses the results and concludes.

2. The Model

Let $X \subseteq \Re^0_+ \times \Re_+$. Each vector $x = (p_x, q_x) \in X$ represents a two-attribute alternative, where $p_x > 0$ is the price and $q_x \geq 0$ some attribute different from price.

On the demand side there are two types of consumer: the fully and the boundedly rational consumer (FRC and BRC, respectively). The FRC is an expected utility maximizer whose Bernoulli utility function is defined as $u(x) = u(g(q_x) - p_x)$, where the function $g(q_x) = \ln(q_x + 1)$ measures how much the FRC values characteristic q_x in monetary units. The FRC is assumed to be risk-averse and her reservation utility is $\bar{u} > u(0)$.

The BRC follows the satisficing heuristic: she judges a good $y \in X$ to be satisfactory whenever $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$, where $\bar{p} > 0$ and $\bar{q} > 0$ are the aspiration levels for price and quality, respectively. Let $(\bar{p}, \bar{q}) \in X$ be the *minimal satisfactory alternative*. The BRC's goal is to get a satisfactory product. If the only available product is unsatisfactory, the BRC walks away without buying anything. If, on the other hand, more than one satisfactory product is available, the BRC chooses the Pareto-dominant one, if it exists. Otherwise, choice is random.

[3] Other studies are Pathak and Sonmez (2008) and Apesteguia and Ballester (2012).

On the supply side, we consider the simplest setting by assuming that there is a risk-neutral monopolist that produces good x whose profit function is $\pi(x) = p_x - \alpha q_x$, where $\alpha \in (0, 1)$. We assume that a consumer not buying anything is the worst possible scenario for the monopolist. The goal of the monopolist is to maximize expected profits.

From now on we denote by $x = (p_x, q_x)$ and $y = (p_y, q_y)$ the products that the monopolist supplies to the FRC and to the BRC, respectively.

3. Analysis

We first assume that the monopolist knows the consumer's type. If the consumer is an FRC, then the monopolist maximizes its profits subject to the reservation-utility constraint (benchmark case).

PROBLEM 1 (benchmark case)
$\max_{p_x,q_x} p_x - \alpha q_x$
s.t. (i) $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$

Let the solution be denoted by $x^c = (p_x^c, q_x^c)$, where c stands for *certainty*. Proofs are relegated in the appendix.

Lemma 1 (Problem 1). *$x^c = (\ln(\frac{1}{\alpha}) - u^{-1}(\bar{u}), \frac{1}{\alpha} - 1)$ and monopolist's profits are $\pi(x^c) = \ln(\frac{1}{\alpha}) - u^{-1}(\bar{u}) - \alpha\left(\frac{1}{\alpha} - 1\right)$.*

Suppose now that the monopolist knows that consumers is a BRC. In this case the monopolist maximizes its profits subject to the constraint that the supplied product y has to be satisfactory, that is, $p_y \leq \bar{p}$ and $q_y \geq \bar{q}$.

PROBLEM 2
$\max_{p_y,q_y} p_y - \alpha q_y$
s.t. (i) $p_y \leq \bar{p}$

(ii) $q_y \geq \bar{q}$

Let the solution be denoted by $y^c = (p_y^c, q_y^c)$. We omit the proof as the result is immediate.

Lemma 2 (Problem 2). *$y^c = (\bar{p}, \bar{q})$ and monopolist's profits are $\pi(y^c) = \bar{p} - \alpha\bar{q}$.*

Let us now consider the interesting case in which the monopolist does not know with certainty whether the consumer is an FRC or a BRC. Let $\rho \in (0, 1)$ be the probability that the consumer is an FRC. We distinguish various cases.

Case (i): $u(x^c) < u(y^c)$. That is, the minimal satisfactory alternative yields more utility than the FRC's optimal alternative under certainty (benchmark case). If y^c yields more utility than x^c, then there is no way in which the monopolist can profitably screen the two types of consumer. The reason is that any satisfactory alternative yields more than the reservation utility. Hence, there is no alternative $z \sim x^c$ that the monopolist can supply in order to induce the FRC to buy z rather than an alternative whose utility is at least $u(\bar{p}, \bar{q})$. This implies that the monopolist supplies a product w whose utility is strictly greater than $\bar{u}$ to both types. Hence, its profits are lower than in the benchmark case.

Case (ii): $u(x^c) \geq u(y^c)$ and x^c and y^c are not Pareto-ranked. In this case the monopolist is forced to screen. The reason is that, on the one hand, the minimal satisfactory alternatives $(\bar{p}, \bar{q})$ yields at most the reservation utility and, on the other hand, the profit-maximizing product that yields the reservation utility is unsatisfactory. Hence, the monopolist supplies x^c to the FRC and y^c to the BRC. Whether profits in this case are higher than in the benchmark case depends on what are the BRC's aspiration levels. In particular, if the minimal satisfactory alternative $(\bar{p}, \bar{q})$ lies on a higher iso-profit curve than $\pi(x^c)$, then profits are higher for any probability $\rho \in (0, 1)$. On the other hand, if the minimal satisfactory alternative is relatively extreme relative to x^c, then it might happen that $\pi(y^c) < \pi(x^c)$ and, therefore, monopolist's profits are lower than in the benchmark case.

Case (iii): x^c Pareto-dominates y^c. That is, $\bar{p} > p_x^c$ and that $\bar{q} < q_x^c$, so that both x^c and y^c are satisfactory, $u(x^c) > u(y^c)$, and unsatisfactory alternatives are relatively 'extreme' from the FRC's point of view. This case is depicted in figure 1.

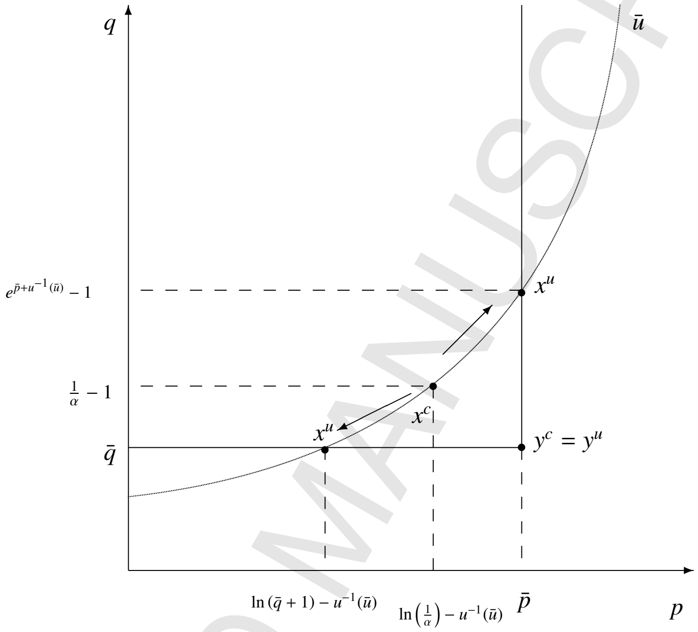

Figure 1: The fact that the BRC does not exchange satisfactory with unsatisfactory alternatives might prevent her from getting something more than $(\bar{p}, \bar{q})$

If the monopolist does not screen, then it is forced to supply x^c to both types. The reason is two-fold. First, the monopolist wants to supply an alternative that yields at least the reservation utility to the FRC. Second, if there is more than one satisfactory alternative available, the BRC buys the Pareto-dominant one. Hence, if the monopolist does not screen, then its profits are those of the benchmark case. That is, $\pi(x^c) = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}) - \alpha\left(\frac{1}{\alpha} - 1\right)$.

It remains to figure out what are the monopolist's profits under the assumption that the monopolist screens.

PROBLEM 3

$\max_{p_x,p_y,q_x,q_y} \rho\,(p_x - \alpha q_x) + (1-\rho)\left(p_y - \alpha q_y\right)$

s.t. (i) $\ln(q_x + 1) - p_x \geq u^{-1}(\bar{u})$

(ii) $p_y \leq \bar{p}$

(iii) $q_y \geq \bar{q}$

(iv) $\ln(q_x + 1) - p_x \geq \ln(q_y + 1) - p_y$

(v) $p_x \geq \bar{p}$ or $q_x \leq \bar{q}$

Problem 3 says that the monopolist has to find two goods $x, y \in X$ such that expected profits are maximized, where x yields the reservation utility and y is satisfactory. However, the optimal solution must satisfy also the incentive-compatible constraints. That is, first, alternative x must yield at least the same level of utility of good y. Second, good x does not have to be more than satisfactory. That is, either $p_x \geq \bar{p}$ or $q_x \leq \bar{q}$.

We denote the solution of problem 3 by x^u and y^u.

Lemma 3 (Problem 3). *Assume that x^c Pareto-dominates y^c.*

1. *If constraint (v) is $p_x \geq \bar{p}$, then $x^u = (\bar{p}, e^{\bar{p}+u^{-1}(\bar{u})} - 1)$, $y^u = (\bar{p}, \bar{q})$, and monopolist's profits are $\pi^u = \rho\left[\bar{p} - \alpha\left(e^{\bar{p}+u^{-1}(\bar{u})} - 1\right)\right] + (1-\rho)\pi(y^c)$.*
2. *If constraint (v) is $q_x \leq \bar{q}$, then $x^u = (\ln(\bar{q}+1) - u^{-1}(\bar{u}), \bar{q})$, $y^u = (\bar{p}, \bar{q})$, and monopolist's profits are $\pi^u = \rho\left[\ln(\bar{q}+1) - u^{-1}(\bar{u}) - \alpha\bar{q}\right] + (1-\rho)\pi(y^c)$.*

The solutions to problem 3 are depicted in figure 1. Assume without loss of generality that imposing constraint $p_x \geq \bar{p}$ is more profitable than imposing $q_x \leq \bar{q}$. Then, monopolist's profits are $\pi^u = \rho\pi(x^u) + (1-\rho)\pi(y^c) = \rho\left[\bar{p} - \alpha\left(e^{\bar{p}+u^{-1}(\bar{u})} - 1\right)\right] + (1-\rho)\pi(y^c)$. The monopolist will screen if and only if $\rho\pi(x^u) + (1-\rho)\pi(y^c) > \pi(x^c)$ or, equivalently, $\rho < \hat{\rho} \equiv \frac{\bar{p} - \left(\ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})\right) + \alpha\left[\left(\frac{1}{\alpha} - 1\right) - \bar{q}\right]}{\alpha\left[\left(e^{\bar{p}+u^{-1}(\bar{u})} - 1\right) - \bar{q}\right]}$.[4] That is, the monopolist will screen if

[4] Note that $\hat{\rho} \in (0, 1)$. The fact that $\hat{\rho} > 0$ follows from the assumption that x^c Pareto-dominates y^c. Next, assume, by contradiction, that $\hat{\rho} > 1$. Then, $\bar{p} - \alpha\left(e^{\bar{p}+u^{-1}(\bar{u})} - 1\right) > \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u}) - \alpha\left(\frac{1}{\alpha} - 1\right)$. Or, equivalently, $\pi(x^u) > \pi(x^c)$. Since both x^c and x^u lie on the indifference curve that yields $\bar{u}$, $\pi(x^u) > \pi(x^c)$ contradicts lemma 1 and the fact that x^c is a profit-maximizing product that yields the reservation utility. Finally, $\hat{\rho} = 1$ if and only if $\bar{p} = p_x^c$, which is ruled out by assumption.

and only if the probability that the consumer is a BRC is sufficiently high. Since $\pi(y^c) > \pi(x^c) > \pi(x^u)$, then a sufficiently high probability that the consumer is a BRC puts more weight on $\pi(y^c)$ relative to $\pi(x^u)$ and therefore makes the screening more profitable.

The next proposition summarizes the observations we have made so far.

Proposition 1 (Welfare).

- *The FRC always enjoys at least the reservation utility $\bar{u}$. If $u(y^c) > u(x^c)$, she enjoys more than $\bar{u}$.*
- *The BRC buys an alternative strictly better than $(\bar{p}, \bar{q})$ if and only if $\rho > \hat{\rho}$ and x^c Pareto-dominates y^c. Otherwise, she gets $(\bar{p}, \bar{q})$.*

Proposition 1 suggests that the FRC clearly benefits from uncertainty. On the other hand, the BRC benefits too, but to a much smaller extent, in the sense that in order for the BRC to get something more than the minimal satisfactory alternative $(\bar{p}, \bar{q})$ not only the probability ρ that the consumer is an FRC has to be sufficiently high, but also the FRC's optimal alternative under certainty (benchmark case) has to Pareto-dominate $(\bar{p}, \bar{q})$. This suggests also that, unlike what one might think, greater rationality on the demand side of the market (i.e., an higher ρ) does not necessarily imply an increase in the consumer's welfare.

4. Discussion and Conclusions

The intuition behind the result of proposition 1 is that the BRC follows a non-compensatory decision strategy, where by noncompensatory we mean a decision strategy according to which the decision-maker avoids trade-offs between attributes (Payne et al., 1993). Indeed, a decision-maker whose behavior is consistent with the satisficing heuristic never exchanges satisfactory with unsatisfactory alternatives.

For an illustration consider again figure 1. In that scenario one expects that under uncertainty the monopolist makes the BRC better off by supplying to both types a good that yields at least the reservation utility $\bar{u}$. However, if the probability that the consumer is an FRC is sufficiently low, then, on the contrary, the monopolist screens the two types of consumer by *pushing* x outside the satisfactory area (towards either north-east or south-west) along the indifference curve that yields $\bar{u}$ until it reaches its border. In this way x^u is unsatisfactory and yields the reservation utility and the monopolist is free to supply the minimal satisfactory alternative to the BRC.[5] Notice that the monopolist can adopt this strategy precisely because, on the one hand, the FRC is perfectly able to make trade-offs between attributes, and, on the other hand, the BRC never exchanges satisfactory with unsatisfactory alternatives.

One way of improving BRC's welfare in the case depicted in figure 1 independently of the probability ρ is to implement policies aimed at making unsatisfactory alternatives unattractive to the FRC. This would diminish the probability that the FRC considers 'extreme' alternatives and, in turn, reduce the monopolist's production space to satisfactory alternatives only. In this case the monopolist would be forced to supply to both types an alternative that yields the reservation utility independently of ρ, making the BRC better off.

Our work can be extended in various ways. First, in the present work we restrict our attention to a limited class of cases (e.g. one specified utility function, two attributes, one good). A natural extension is to perform the above analysis by considering a more general framework. Second, in the proposed model we assume that the monopolist knows the BRC's aspiration levels. An interesting extension would be to investigate whether the results of proposition 1 are robust to the relaxation of this assumption. We leave this for future research.

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[5]Strictly speaking, x^u is satisfactory. However, since we are on the continuum, we interpret x^u as unsatisfactory, where either price or quality do not meet the aspiration level.

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Abdulkadiroglu, A., Che, Y.-K., Yasuda, Y., 2011. Resolving conflicting preferences in school choice: The 'boston mechanism' reconsidered. American Economic Review 101, 399–410.

Apesteguia, J., Ballester, M. A., 2012. Naive and sophisticated players in school choice. Economics Letters 115 (2), 172–174.

Caplin, A., Dean, M., 2011. Search, choice, and revealed preference. Theoretical Economics 6, 19–48.

Caplin, A., Dean, M., Martin, D., 2011. Search and satisficing. American Economic Review 101 (7), 2899–2922.

Ellison, G., 2006. Bounded rationality in industrial organization. In: Blundell, R., Newey, W., Persson, T. (Eds.), Advances in Economics and Econometrics: Theory and Applications. Cambridge University Press.

Horan, S., 2010. Sequential search and choice from lists. Boston University. Job Market Paper.

Papi, M., 2012. Satisficing choice procedures. Journal of Economic Behavior and Organization 84, 451–462.

Pathak, P. A., Sonmez, T., 2008. Leveling the playing field: Sincere and sophisticated players in the boston mechanism. American Economic Review 98, 1636–1652.

Payne, J. W., Bettman, J. R., Johnson, E. J., 1993. The Adaptive Decision Maker. Cambridge University Press.

Reutskaja, E., Nagel, R., Camerer, C. F., Rangel, A., 2011. Search dynamics in consumer choice under time pressure: An eye-tracking study. American Economic Review 101, 900–926.

Rubinstein, A., Salant, Y., 2006. A model of choice from lists. Theoretical Economics 1 (1), 3–17.

Simon, H. A., 1955. A behavioral model of rational choice. Quarterly Journal of Economics 69 (1), 99–118.

Spiegler, R., 2006. Competition over agents with boundedly rational expectations. Theoretical Economics 1, 207–231.

Spiegler, R., 2011. Bounded Rationality and Industrial Organization. Oxford University Press.

Tyson, C. J., 2008. Cognitive constraints, contraction consistency, and the satisficing criterion. Journal of Economic Theory, 138, 51–70.

AppendixA. Proofs

Proof of Proposition 1. Problem 1 can be rewritten as follows.
$\max_{p_x,q_x} p_x - \alpha q_x$
s.t. (i) $p_x - \ln(q_x + 1) \leq -u^{-1}(\bar{u})$

Here are the Lagrangian and the Kuhn-Tucker conditions.

$\mathcal{L}(p_x, q_x, \lambda) = p_x - \alpha q_x - \lambda(p_x - \ln(q_x + 1) + u^{-1}(\bar{u}))$

$$\frac{\partial \mathcal{L}}{\partial p_x} = 1 - \lambda = 0 \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial q_x} = -\alpha + \frac{\lambda}{q_x + 1} \leq 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \ln(q_x + 1) - p_x - u^{-1}(\bar{u}) \geq 0 \tag{A.3}$$

together with $p_x > 0$, $q_x, \lambda \geq 0$, $q_x\left(\frac{\partial \mathcal{L}}{\partial q_x}\right) = 0$, and $\lambda\left(\frac{\partial \mathcal{L}}{\partial \lambda}\right) = 0$.

By condition A.1, $\lambda^* = 1$. Therefore, constraint (i) must be binding. Next, suppose, by contradiction, that $q_x = 0$. This implies that $p_x = -u^{-1}(\bar{u}) < 0$, which leads to a contradiction. Therefore, $q_x > 0$ and condition A.2 must be binding. Plugging $\lambda^* = 1$ into condition A.2 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.3 and solving for p_x, we get $p_x^* = \ln(\frac{1}{\alpha}) - u^{-1}(\bar{u})$. Let this solution be denoted by x^c.■

Proof of Lemma 3. We prove the statement in the case in which constraint (v) is $\bar{p} \leq p_x$. The case in which constraint (v) is instead $q \leq \bar{q}$ is analogous and, therefore, omitted. We rewrite below problem 3.

$\max_{p_x, p_y, q_x, q_y} \rho\,(p_x - \alpha q_x) + (1-\rho)\left(p_y - \alpha q_y\right)$
s.t. (i) $p_x - \ln(q_1 + 1) \leq -u^{-1}(\bar{u})$
(ii) $p_y - \bar{p} \leq 0$
(iii) $\bar{q} - q_y \leq 0$
(iv) $p_x - \ln(q_x + 1) - p_y + \ln(q_y + 1) \leq 0$
(v) $\bar{p} - p_x \leq 0$

Here are the Lagrangian and the Kuhn-Tucker conditions.

$\mathcal{L}(p_x, p_y, q_x, q_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) = \rho\,(p_x - \alpha q_x) + (1-\rho)\left(p_y - \alpha q_y\right) - \lambda_1(p_x - \ln(q_x+1) + u^{-1}(\bar{u})) - \lambda_2(p_y - \bar{p}) - \lambda_3(\bar{q} - q_y) - \lambda_4(p_x - \ln(q_x+1) - p_y + \ln(q_y+1)) - \lambda_5(\bar{q} - p_x)$

$$\frac{\partial \mathcal{L}}{\partial p_x} = \rho - \lambda_1 - \lambda_4 + \lambda_5 = 0 \quad \text{(A.4)}$$
$$\frac{\partial \mathcal{L}}{\partial p_y} = (1-\rho) - \lambda_2 + \lambda_4 = 0 \quad \text{(A.5)}$$
$$\frac{\partial \mathcal{L}}{\partial q_x} = -\rho\alpha + \frac{\lambda_1}{q_x+1} + \frac{\lambda_4}{q_x+1} \leq 0 \quad \text{(A.6)}$$
$$\frac{\partial \mathcal{L}}{\partial q_y} = -(1-\rho)\alpha + \lambda_3 - \frac{\lambda_4}{q_y+1} \leq 0 \quad \text{(A.7)}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \ln(q_x+1) - p_x - u^{-1}(\bar{u}) \geq 0 \quad \text{(A.8)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \bar{p} - p_y \geq 0 \tag{A.9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = q_y - \bar{q} \geq 0 \tag{A.10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_4} = \ln(q_x + 1) - p_x - \ln(q_y + 1) + p_y \geq 0 \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_5} = p_x - \bar{p} \geq 0 \tag{A.12}$$

together with $p_x, p_y > 0$, $q_x, q_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \geq 0$, $q_x \left(\frac{\partial \mathcal{L}}{\partial q_x}\right) = 0$, $q_y \left(\frac{\partial \mathcal{L}}{\partial q_y}\right) = 0$, $\lambda_1 \left(\frac{\partial \mathcal{L}}{\partial \lambda_1}\right) = 0$, $\lambda_2 \left(\frac{\partial \mathcal{L}}{\partial \lambda_2}\right) = 0$, $\lambda_3 \left(\frac{\partial \mathcal{L}}{\partial \lambda_3}\right) = 0$, $\lambda_4 \left(\frac{\partial \mathcal{L}}{\partial \lambda_4}\right) = 0$, and $\lambda_5 \left(\frac{\partial \mathcal{L}}{\partial \lambda_5}\right) = 0$.

Notice that since $\bar{q} > 0$, then, by condition A.10, $q_y > 0$, and, therefore, condition A.7 must be binding. Therefore, it must be that $\lambda_3 > 0$. This implies that $q_y^* = \bar{q}$. Next, condition A.5 implies that $\lambda_2 > 0$. Hence, $p_y = \bar{p}$. Further, assume by contradiction that $q_x = 0$. Then, by condition A.8, $p_x \leq -u^{-1}(\bar{u})$. Since $u^{-1}(\bar{u}) > 0$, then $p_x < 0$, which leads to a contradiction. Therefore, $q_x > 0$ and condition A.6 must be binding. Finally, notice that it cannot be that $\lambda_1 = \lambda_4 = 0$, because otherwise, by condition A.4, $\lambda_5 = -\rho$, which leads to a contradiction.

- Assume first that $\lambda_5 = 0$. This implies that $p_x > \bar{p}$.
  - Suppose that $\lambda_1, \lambda_4 > 0$. Hence, constraints (i) and (iv) must be binding. Moreover, by condition A.4, $\lambda_1 + \lambda_4 = 1$. Using this result in condition A.6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.8 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})$. However, this contradicts the fact that x^c Pareto-dominates y^c.
  - Assume $\lambda_1 > 0$ and $\lambda_4 = 0$. This implies that constraint (i) must be binding and constraint (iv) holds with strict inequality. Moreover, by condition A.4, $\lambda_1 = \rho$. Using this result in condition A.6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.8 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - u^{-1}(\bar{u})$. However, again this contradicts the fact that x^c Pareto-dominates y^c.
  - Assume that $\lambda_1 = 0$ and $\lambda_4 > 0$. This implies that constraint (i) holds with strict inequality and constraint (iv) must be binding. Moreover, by

condition A.4, $\lambda_4 = \rho$. Using this result in condition A.6 and solving for q_x, we get $q_x^* = \frac{1}{\alpha} - 1$. Next, plugging q_x^* into condition A.11 and solving for p_x, we get $p_x^* = \ln\left(\frac{1}{\alpha}\right) - \ln(\bar{q} + 1) + \bar{p}$. Again, a contradiction.

- Suppose that $\lambda_5 > 0$. This implies that $p_x^* = \bar{p}$.
  - Suppose that $\lambda_1, \lambda_4 > 0$. This implies that constraints (i) and (iv) must be binding. Plugging p_x^* into condition A.8 and solving for q_x, we get $q_x^* = e^{\bar{p}+u^{-1}(\bar{u})} - 1$. Next, by condition A.11, $\bar{p} + u^{-1}(\bar{u}) - \bar{p} = \ln(\bar{q} + 1) - \bar{p}$, which implies that $\bar{u} = u(\bar{p}, \bar{q})$. However, this contradicts the fact that x^c Pareto-dominates y^c.
  - Suppose that $\lambda_1 > 0$ and $\lambda_4 = 0$. This implies that constraint (i) must be binding and constraint (iv) holds with strict inequality. Plugging p_x^* into condition A.8 and solving for q_x, we get $q_x^* = e^{\bar{p}+u^{-1}(\bar{u})} - 1$. Next, by condition A.11 $\bar{p}+u^{-1}(\bar{u})-\bar{p} > \ln(\bar{q}+1)-\bar{p}$, which implies that $\bar{u} > u(\bar{p}, \bar{q})$.
  - Suppose that $\lambda_1 = 0$ and $\lambda_4 > 0$. This implies that constraint (i) holds with strict inequality and constraint (iv) must be binding. Plugging p_x^* into condition A.11 and solving for q_x, we get $q_x^* = \bar{q}$. Next, by condition A.8 $\ln(\bar{q} + 1) - \bar{p} - u^{-1}(\bar{u}) > 0$, which implies that $\bar{u} < u(\bar{p}, \bar{q})$. Again this contradicts the fact that x^c Pareto-dominates y^c.

The only case in which a contradiction does not take place is when $p_x^* = \bar{p}$, $q_x^* = e^{\bar{p}+u^{-1}(\bar{u})} - 1$. Note that in this case monopolist's profits are $\rho\left[\bar{p} - \alpha\left(e^{\bar{p}+u^{-1}(\bar{u})} - 1\right)\right] + (1 - \rho)\pi(y^c)$. This concludes the proof.■

> A monopolist supplies a multi-attribute good. > The monopolist does not know whether the consumer is a maximizing or satisficing type. > The satisficing type almost never benefits from uncertainty.

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