UNIVERSITY OF ABERDEEN: SESSION 2022-2023

Degree Examination in PX3512 **Electricity and Magnetism**

10th May 2023 **3:00-5:00 pm**

Candidates should attempt **ALL** questions in Section A — 35 marks
and **TWO** questions from Section B — 40 marks
Total 75 marks

Formulae and data

Gauss' theorem in vector analysis: $\oint \boldsymbol{v}d\boldsymbol{a} = \int \boldsymbol{\nabla v}\, d\tau$
Stokes' theorem in vector analysis: $\oint \boldsymbol{v}d\boldsymbol{l} = \int \boldsymbol{\nabla}\text{x}\boldsymbol{v}\, d\boldsymbol{a}$

Maxwell's equations:

Integral form	Differential form
$\oint \boldsymbol{E}d\boldsymbol{a} = \frac{1}{\varepsilon_0}\int \rho d\tau$	$\nabla.\mathbf{E} = \frac{\rho}{\varepsilon_0}$
$\oint \boldsymbol{B}d\boldsymbol{a} = 0$	$\nabla.\mathbf{B} = 0$
$\oint \boldsymbol{E}d\boldsymbol{l} = -\int \frac{\partial \boldsymbol{B}}{\partial t}d\boldsymbol{a}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\oint \boldsymbol{B}d\boldsymbol{l} = \mu_0 \int (\boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t})d\boldsymbol{a}$	$\nabla \times \mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$

Potentials:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r} \qquad \mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}\, d\tau}{r}$$

Energy densities:

$$U_E = \tfrac{1}{2}\mathbf{E.D} \qquad U_B = \tfrac{1}{2}\mathbf{B.H}$$

Poynting vector: $\boldsymbol{N} = \boldsymbol{E}\ x\ \boldsymbol{H}$

$e = 1.6\times10^{-19}$ C
$\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}
$\mu_0 = 4\pi\ 10^{-7}$ H m^{-1}
$c = 3.0\times10^{8}$ m s^{-1}

SECTION A : Answer ALL questions (Total marks: 35)

1) Calculate the frequency at which we have to rotate a conducting loop consisting of N=500 turns in the presence of a fixed external magnetic field of magnitude 0.2 T to generate a maximal electromotive force of 1V. The cross-section area of 1 turn of the loop is equal to 100cm^2 (see Fig.1).

[10]

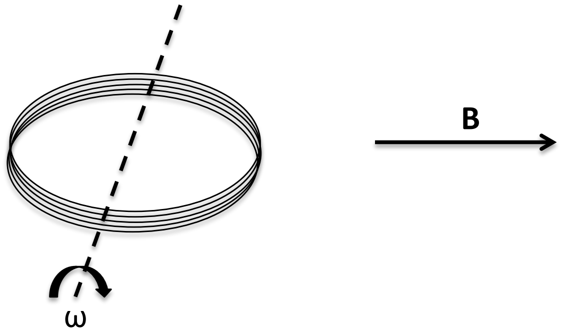


Figure 1

2) A typical construction for an electromagnet is a coil filled with magnetic material, with a gap, where we generate a magnetic field by passing a current through the coil (see Figure 2). The region inside the coil depicted in Figure 2 is filled with a magnetic material of relative permeability μ_r=1000, the width of the gap is equal to w=0.2m and the radius of the toroid is r=6m. The total number of turns of the coil is equal to N=500.

a) Calculate the magnitude of the current running through the coil of the electromagnet needed to generate a magnetic field of 0.1T in the gap. [7]
b) Calculate the magnetisation of the material inside the coil. [3]

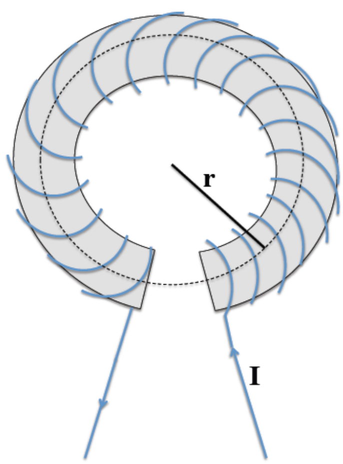


Figure 2

3) A plane electromagnetic wave travels in the x-direction in vacuum, and is polarised with its electric field E in the z-direction. Its average energy intensity is equal to 8 mW/m^2 and it has a frequency f=200 MHz.
   a) Calculate the amplitude of the emf induced in a single turn loop of radius 5cm lying on the x-z plane. [3]
   b) Calculate the amplitude of the emf induced if the loop lies on the x-y plane. [2]

4) Show that each Cartesian component of the electric field in a vacuum satisfies the wave equation.

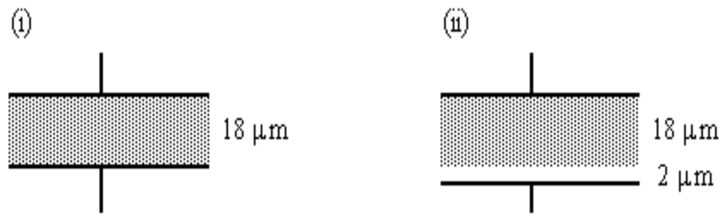
   Hint: $\nabla \times (\nabla \times \vec{a}) = \nabla(\nabla \vec{a}) - \nabla^2 \vec{a}$ [7]

5) Use Gauss's law to calculate the electric flux $\oint \boldsymbol{E d a}$ through the following closed surfaces, all three of them containing a charge of Q Coulombs in the centre: i) a sphere, ii) a cube, iii) a cylinder.
   [3]

SECTION B : Answer TWO questions (Total marks: 40)

6)
   (a) Define the electric polarisation, **P**. [2]
   (b) Explain what is meant by *bound charge* and how the bound charge density, $\rho_b = -\nabla.\mathbf{P}$, leads to the introduction of the electric displacement vector, **D**. [6]
   (c) Calculate the breakdown voltages of the two parallel plate capacitors described below.
      (i) The plate separation is 18μm and the space is filled with an insulator of relative permittivity 26.0 and dielectric strength 470 kVmm^{-1}.

      ***Dielectric strength** is the maximal electric field strength that an insulator can withstand intrinsically without breaking down, i.e., without experiencing failure of its insulating properties.* [4]
      (ii) The same as (i) except that an additional 2μm thick layer of air has been inadvertently introduced during manufacture. Air has a relative permittivity of 1.0 and dielectric strength 3 kVmm^{-1}. [8]



7) For the toroidal magnetic circuit shown in Fig.3 (next page), R=4 cm, g=1 mm, the cross-section area is $A=4\ cm^2$, $N_1=300$ turns, $N_2=200$ turns, and the material inside the former is a linear magnetic material with $\mu_r=500$.

(a) Calculate the following:
   (i) The self-inductance of coil 1. [5]
   (ii) The self-inductance of coil 2. [5]
   (iii) The mutual inductance between the coils. [5]

(b) Calculate the corresponding values if there are no gaps, i.e., g=0. [5]

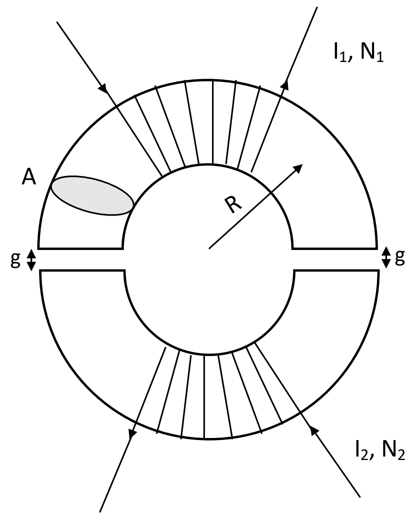


Figure 3

8) An air spaced parallel plate capacitor with plate spacing 0.5mm and capacitance 10μF stores the energy that a laser system needs to generate a pulse of light. The energy stored in the capacitor is converted to light with efficiency of 0.5%. The laser delivers a 20kW pulse of duration 5μs and beam diameter 2mm. The light beam may be treated as a plane wave of wavelength $\lambda = 600$nm.
Calculate:

(a) the voltage to which the capacitor is charged. [5]

(b) the energy density in the electric field of the capacitor. [5]

(c) the maximum value of the electric field in the light pulse. [5]

(d) the values of the parameters B_0, ω and k in the expression for the magnetic field:
$$B(z,t) = B_0 \sin(\omega t - kz)$$ [5]

Hint: The total energy stored in a capacitor is equal to $CV^2/2$.

THE END