UNIVERSITY OF ABERDEEN: SESSION 2022/2023

Degree Examination in PX*3014* ***Energy and Matter***

8th December 2022 9.00 - 11.00 am

PLEASE READ CAREFULLY

Answer all question in section A and three question in section B

You may need to use the following constants in your calculations:

Constant	**symbol**	**value**
Boltzmann's constant	k_B	$= 1.38 \times 10^{-23}\ J\ K^{-1}$
Electron Charge	e	$= 1.6 \times 10^{-19}$ *Coulombs*
Avogadro's Number	N_A	$= 6.022 \times 10^{23}\ mole^{-1}$

Reciprocity theorem: $(\partial f/\partial x)_y = -(\partial f/\partial y)_x(\partial y/\partial x)_f$

SECTION A ***answer all questions.***

1. Determine an expression for the contact angle of a liquid drop on a solid surface in terms of the surface energies of the liquid, solid and liquid solid interface (draw a sketch showing the forces acting on the drop). What is a surfactant? Give an example. How would it change this angle? **[5]**

2. The Bolzmann distribution states that the probability p_i of an energy state E_i is $p_i \propto \exp(-E_i / k_B T)$ where T is the temperature of the system. For particles of radius a and density ρ suspended in a fluid of density ρ_o determine the relative density of particles at a height h compared to the base of the vessel. Comment on the implications for this in terms of the particle size and whether particles will settle or not. Give an example of how you can force particles to settle in the case of blood cells. **[5]** .

3. Consider a block of material with mass m and specific heat capacity C which changes in temperature from T_1 to T_2 . By considering the relationship between entropy, heat energy and temperature show that the change of entropy in the block is $\Delta S = mC\log_e(T_2/T_1)$. **[5]**

4. Assuming that $(1/P)(\partial P/\partial V)_S = -\gamma/V$ where P is pressure, V volume and γ the ratio of the constant pressure and constant volume heat capacities, show that, for an adiabatic process, $PV^\gamma =$ constant and write down an expression for the work done in an adiabatic expansion/compression. **[5]**

SECTION B *Answer three question only*

5. (a) Using a sketch to illustrate your answer, consider a pipe of radius, *a*, and length, *L*, with a pressure difference, Δp, across its ends. (i) Consider and include in your sketch a cylindrical region of fluid of radius, *r*, within the tube which is coaxial with it. (ii) State clearly what the forces acting on this cylinder are. (iii) Hence determine that (for laminar flow) the velocity v as a function of radius r is: **[5]**

$$(dv/dr) = -(r\Delta P)/(2L\eta)$$

(b) Now derive an expression for the velocity of the fluid at a radius r stating your assumption about the boundary conditions. **[5]**

(c) Using a sketch to illustrate your argument, determine the volume of fluid flowing through the pipe each second. **[5]**

(d) Pumping fluid through a pipe too fast will cause turbulent flow. The velocity above which this type of flow sets in is a function of the density of the fluid, its visosity and radius of the pipe. Determine, using dimensional analysis, what this relationship is (dimensions of viscosity are $ML^{-1}T^{-1}$). **[5]**

6 (a) Prove that the energy density of a stretched material is $Y\varepsilon^2/2$ where *Y* is the Young's modulus and ε is the strain. Argue that the theoretical breaking stress σ_B of a material ought to be $\sigma_B = 2(\gamma Y/r)^{1/2}$ where γ is the surface tension of the material and *r* the interatomic distance. Is this value realistic? Explain your answer and indicate how the equation can be changed to take account of other factors. **[6]**

Figure 1 A failed connecting rod

(b) Figure 1 shows a connecting rod, with the help of a sketch, show where the failure started and how it developed. Discuss how swords, steel, mud bricks and fibre glass canoes can be engineered to improve their strength, hardness and resistance to failure. **[8]**

(c) Give details of the Comet aeroplane disasters and explain how key engineering decisions led to the catastrophic failures **[6]**

7. (a) Define, in words and algebraically the specific heat capacities C of a gas at (a) constant pressure and (b) constant volume in terms of the temperature T and entropy. Which would you expect to be greater? Qualitatively justify your answer . **[6]**

(b) Write down a relationship for changes in entropy at constant pressure (ie with respect to volume and temperature). Using this relationship and the Maxwell relationship (below) determine how C_p and C_v are related to each other. **[8]**

$$\left(\frac{\delta S}{\delta V}\right)_T = \left(\frac{\delta p}{\delta T}\right)_V$$

(c) Use the differential form of the Helmholtz function (below) to verify the above Maxwell relationship. **[6]**

$$\delta F = -S\delta T - P\delta V$$

8. (a) What is the second law of thermodynamics? Sketch a graph of how *S* changes with increasing energy and hence use this to define thermodynamic temperature *T*. **[4]**

(b) Draw a *PV* diagram illustrating the cycle of an idealized reversible heat engine. How much work is done in each cycle – justify your answer. Explain the changes in entropy in the system throughout the cycle derive an expression for the efficiency of the heat engine in terms of the temperatures of the source and sink. **[10]**

(c) Using sketches, discuss the evolution and efficiencies of the steam engine from the early 18th century. Discuss the technological innovations with reference to some of the principal inventors involved in these developments. **[6]**

THE END