Passive Bridge Deck-flaps System for Suppression of Flutter in Long Span Bridges

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ABSTRACT

Traditional passive control techniques cannot satisfactorily suppress flutter in ultra long-span bridges. In this paper an aerodynamic flutter control by additional flaps attached directly to the bridge deck is proposed. State-space equation of motion is derived through rational function approximation of unsteady aerodynamic forces due to the Theodorsen solution for wing-aileron-tab combination. Control flaps motion is assumed proportional to the pitching motion of the deck. Two practical implementations of the control system are proposed. The first improves critical wind velocity by 120 %, but requires changes in its configuration due to changes in wind direction. The second system improves critical wind speed by 45 %, but employs flaps of small size and is independent from wind direction.

1. INTRODUCTION

Structure-wind interaction phenomena, static as well as dynamic, are becoming increasingly important as spans become longer and bridge girders more flexible. Flutter instability, in case of ultra long-span bridges, is most often a design governing criterion, since it may lead to the total collapse of a bridge structure.

After Tacoma Narrows Bridge failure the rebuilt bridge employed very deep, torsionally rigid truss girder to safeguard the structure against flutter. In order to overcome the drawbacks of truss cross-section (Ostenfeld and Larsen, 1992) the concept of flat box girder was introduced into design and construction of Severn suspension bridge. However, flat box girders are prone to coupled flutter and supplementary suppression techniques must be considered. Various methods, such as Tuned Mass Damper (Nobuto et al., 1988) or deployment of eccentric mass (Branceleoni, 1992), were considered, but satisfactory solution has not been found yet.

The aerodynamic control of bridge flutter by additional surfaces attached beneath the deck was proposed by Ostenfeld and Larsen (1992). In their concept the rotational displacement of the control surfaces is actively adjusted by feedback control in order to generate aerodynamic forces stabilizing the deck. This system was studied experimentally by Kobayashi and Nagaoka (1992) and theoretically by Wilde and Fujino (1998).

The control system components should be duplicate or triplicate in the form of parallel systems, and must be constantly monitored and maintained. This would lead to high costs of active control systems. Wilde et al. (1998) proposed the concept of passive system utilizing control surfaces. In their study control surfaces motion was govern by an additional pendulum attached to the center of gravity of the deck.

Aerodynamic control may be also done by additional flaps attached directly to the edges of the bridge deck. In this system the flow pattern around the deck is affected by the motion of the flaps and thus, the stabilizing action comes not only from aerodynamic forces generated on control flaps but also can be achieved through modification of aerodynamic forces exerted on the bridge deck.

The aim of this study is to formulate a mathematical model of the passive aerodynamic control by bridge deck-flaps system and propose a configuration of passive control system.

2. MODELING OF BRIDGE DECK - FLAPS SYSTEM

A theoretical description of the self-excited aerodynamic forces of a wing-aileron-tab combination was derived from potential flow theory by Theodorsen and Garrick (1943). Roger (1977) proposed a modeling method which can transform the aeroelastic equation of motion of an airplane into time domain. This method approximates aerodynamic force coefficients by rational functions of the Laplace variable. The application of the rational function approximation for flutter analysis of bridges was reported by Wilde et al. (1996).

2.1. Equation of Motion

A section of a bridge deck-flaps system (Fig. 1) is assumed to have four degrees of freedom: vertical and torsional displacement with respect to the elastic center of the deck, denoted by h, α and referred to as *heaving* and *pitching* motion, respectively, and relative rotation of leading and trailing control flap, denoted as β and γ. The deck together with flaps has the chord width of $2b$. For small amplitude oscillations the total aerodynamic force can be assumed to be a superposition of self-excited forces $\mathbf{P}$ and buffeting forces $\mathbf{P}_{buf}$. The equation of motion can be written in matrix form:

$$\mathbf{M}_s\ddot{\mathbf{x}} + \mathbf{C}_s\dot{\mathbf{x}} + \mathbf{K}_s\mathbf{x} = \mathbf{P} + \mathbf{P}_{buf} \qquad (1)$$

$\mathbf{M}_s$, $\mathbf{C}_s$, and $\mathbf{K}_s$ are system mass, damping, and stiffness matrices, respectively. Vectors of displacements and corresponding self-exited forces are selected as:

$$\mathbf{x}' = [h/b \quad \alpha \quad \beta \quad \gamma], \qquad \mathbf{P}' = [L_h b \quad M_\alpha \quad M_\beta \quad M_\gamma] \qquad (2a, b)$$

where L_h is a lift force, M_α, M_β, and M_γ are torsional moments acting on corresponding degrees of freedom.

The unsteady aerodynamic forces acting on a wing-aileron-tab combination (Fig. 2), according to Theodorsen and Garrick (1943), may be represented as

$$\mathbf{F} = \mathbf{M}_a\ddot{\mathbf{q}} + (U/b)\mathbf{C}_a\dot{\mathbf{q}} + (U/b)^2\mathbf{K}_a\mathbf{q} + C(\hat{s})(U/b)\mathbf{R}\mathbf{S}_2\dot{\mathbf{q}} + C(\hat{s})(U/b)^2\mathbf{R}\mathbf{S}_1\mathbf{q} \qquad (3)$$

where vectors of displacements and corresponding self-exited forces are:

$$\mathbf{q}' = \begin{bmatrix} q_h/b & q_\alpha & q_\beta & q_\gamma \end{bmatrix}, \qquad \mathbf{F}' = \begin{bmatrix} L_{qh}b & M_{q\alpha} & M_{q\beta} & M_{q\gamma} \end{bmatrix} \qquad (4a, b)$$

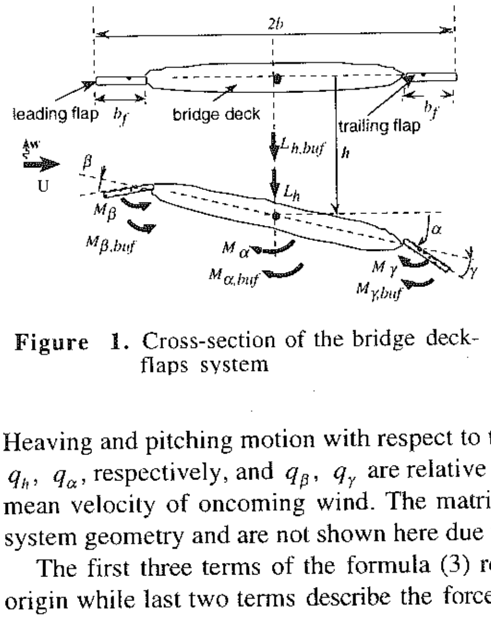

Figure 1. Cross-section of the bridge deck-flaps system

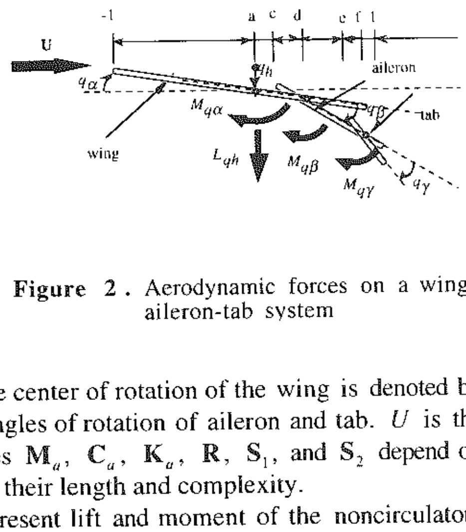

Figure 2. Aerodynamic forces on a wing-aileron-tab system

Heaving and pitching motion with respect to the center of rotation of the wing is denoted by q_h, q_α, respectively, and q_β, q_γ are relative angles of rotation of aileron and tab. U is the mean velocity of oncoming wind. The matrices $\mathbf{M}_a$, $\mathbf{C}_a$, $\mathbf{K}_a$, $\mathbf{R}$, $\mathbf{S}_1$, and $\mathbf{S}_2$ depend on system geometry and are not shown here due to their length and complexity.

The first three terms of the formula (3) represent lift and moment of the noncirculatory origin while last two terms describe the forces due to the vorticity in the wake generated at the trailing edge.

Vectors of displacements and self-exited aerodynamic forces describing the bridge deck-flaps system and wing-aileron-tab combination are related through linear transformations

$$\mathbf{q} = \mathbf{V}\mathbf{x}, \qquad \mathbf{P} = \mathbf{V}'\mathbf{F} \qquad (5a, b)$$

The function $C(\hat{s})$ appearing in (3) is the generalized Theodorsen function (Edwards, 1977) which is a function of nondimensional Laplace variable $\hat{s}$.

2.2. Rational Function Approximation (RFA)

The generalized Theodorsen function is approximated by rational functions of the nondimensionalized Laplace variable as

$$\tilde{C}(\hat{s}) = a_0 + \sum_{i=1}^{n_l} a_i/(\hat{s} + b_i) \quad (6)$$

where $\tilde{C}(\hat{s})$ denotes approximation. The partial fractions, $a_i/(\hat{s}+b_i)$, are called *lag terms*. The coefficients of the partial fractions b_i are referred to as *lag coefficients*. Addition of each partial fraction introduces into the resulting state-space realization new states referred to as *aerodynamic states*. The number of partial fractions is denoted by n_l and this number is found as a compromise between the precision of the approximation and the size of state-space realization.

Applying coordinate transformation (5) to aerodynamic forces (3) and taking Laplace transform with the approximated formula for the Theodorsen function (6) and zero initial condition yields self-exited forces for deck-flaps system in Laplace domain

$$\begin{aligned}\mathcal{L}(\mathbf{P}) = \mathbf{V}'\Big\{\mathbf{M}_a s^2 + (U/b)(\mathbf{C}_a + a_0\mathbf{R}\mathbf{S}_2)s + (U/b)^2(\mathbf{K}_a + a_0\mathbf{R}\mathbf{S}_1) \\ +(U/b)^2\mathbf{R}\sum_{i=1}^{n_l}\{a_i/[s+(U/b)b_i]\}[\mathbf{S}_2 s + (U/b)\mathbf{S}_1]\Big\}\mathbf{V}\mathcal{L}(\mathbf{x})\end{aligned} \quad (7)$$

The aerodynamic states are defined in Laplace domain as

$$\mathcal{L}(\mathbf{x}_a) = [s\mathbf{I} - (U/b)\mathbf{R}_b]^{-1}(U/b)\mathbf{E}\mathcal{L}(\mathbf{x}) \quad (8)$$

The matrices appearing in (8) have the following forms

$$\mathbf{E} = -[b_1,\dots,b_{n_l}]'\mathbf{S}_2\mathbf{V} + [\underbrace{1,\dots,1}_{n_l\ times}]'\mathbf{S}_1\mathbf{V}, \qquad \mathbf{R}_b = -diag(b_1,\dots,b_{n_l}) \quad (9a, b)$$

Taking the inverse Laplace transform of (7) and (8), and inserting them into equation of motion (1) yields a state space equation of motion

$$\begin{bmatrix}\dot{\mathbf{x}}\\ \ddot{\mathbf{x}}\\ \dot{\mathbf{x}}_a\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} & \mathbf{M}^{-1}\mathbf{Q}\\ (U/b)\mathbf{E} & \mathbf{0} & (U/b)\mathbf{R}_b\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\\ \mathbf{x}_a\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{P}_{buff}\\ \mathbf{0}\end{bmatrix} \quad (10a)$$

where

$$\mathbf{M} = \mathbf{M}_s - \mathbf{V}'\mathbf{M}_a\mathbf{V},\ \ \mathbf{K} = \mathbf{K}_s - (U/b)^2\mathbf{V}'\Big(\mathbf{K}_a + a_0\mathbf{R}\mathbf{S}_1 + \sum_{i=1}^{n_l} a_i\mathbf{R}\mathbf{S}_2\Big)\mathbf{V} \quad (10b, c)$$

$$\mathbf{C} = \mathbf{C}_s - (U/b)\mathbf{V}'(\mathbf{C}_a + a_0\mathbf{R}\mathbf{S}_2)\mathbf{V}, \qquad \mathbf{Q} = (U/b)^2\mathbf{V}'\mathbf{R}[a_1,\dots,a_{n_l}] \quad (10d, e)$$

The augmented state vector contains aerodynamic states, $\mathbf{x_a}$. In this RFA formulation addition of one lag term results in addition of only one new aerodynamic state.

2.3. Verification of RFA Model

Time domain modeling of unsteady aerodynamics of bridge deck, described in the previous section, is applied to the analysis of sectional model of a suspension bridge with main span

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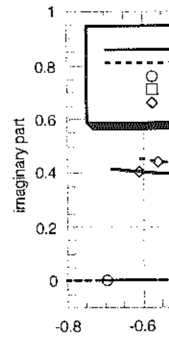

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of 2000 m and streamlined deck of width $2b = 30\ m$. Undamped circular frequencies of this model are $\omega_h = 0.389\ rad/s$ and $\omega_\alpha = 0.892\ rad/s$ for heaving and pitching mode, respectively (Wilde and Fujino, 1998).

The formula for approximation of the Theodorsen function was found by curve fitting of the exact function $C(\hat{s})$ with its approximation $\tilde{C}(\hat{s})$ for the argument $\hat{s} = ik$, in the interval of the reduced frequencies from $k_{low} = 0.07$ up to $k_{up} = 1.5$, which covers the bridge response at the wind speed from 10 m/s up to 100 m/s. The approximation with 2 lag terms provides good compromise between the accuracy of the fit and the number of lags, and thus, is herein employed:

$$\tilde{C}(\hat{s}) = 0.504 + 0.031/(\hat{s} + 0.117) + 0.081/(\hat{s} + 0.432) \quad (11)$$

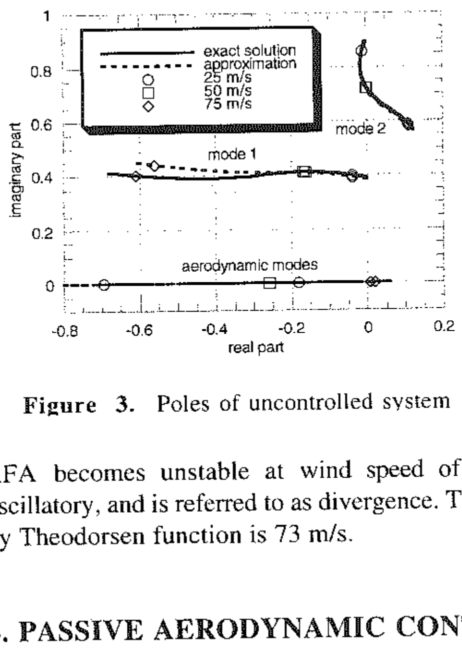

Figure 3. Poles of uncontrolled system

Eigenvalue analysis of the state-space equation yields poles of the system. Their variation for wind speed from 0 m/s up to 80 m/s, together with values obtained from standard eigenvalue analysis for the system with aerodynamic forces computed with the Theodorsen function are shown in Fig. 3. It can be seen that poles of the system for approximated and exact formulation of the Theodorsen function are in good agreement. The flutter wind speed, for both methods, is found to be 50 m/s. At this wind speed the real part of eigenvalue corresponding to the mode 2 (pitching dominant at low wind velocity) becomes positive. The aerodynamic pole of the bridge model with RFA becomes unstable at wind speed of 74 m/s. The corresponding motion is non-oscillatory, and is referred to as divergence. The divergence wind speed for the system defined by Theodorsen function is 73 m/s.

4. PASSIVE AERODYNAMIC CONTROL

A proposed passive control system (Fig. 4) consists of auxiliary flaps attached directly to the bridge deck. When the deck undergoes pitching motion, control flaps rotation is forced by additional cables connecting control flaps and the point lying in the mid-distance between the main cables of the bridge. This point is located on a transverse beam connecting the main cables of the bridge. Since cables can only pull the flaps, additional prestressed springs are used to force appropriate motion of the control surfaces. The system shown in Fig. 4a can work properly only for wind coming from one direction and requires alteration of its configuration as wind direction changes. The performance of the system shown in Fig. 4b is, on the contrary, independent from wind direction.

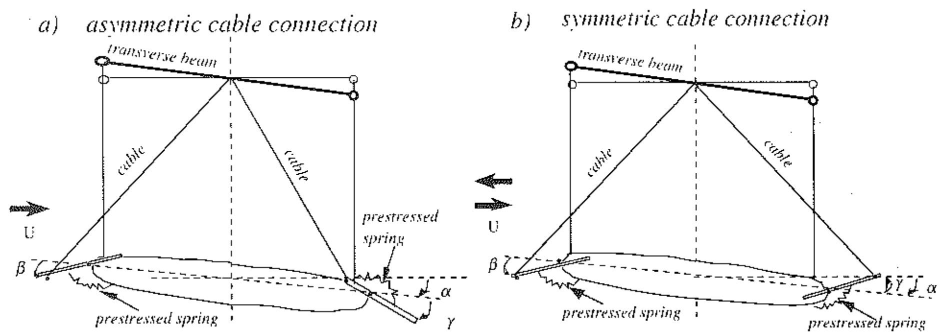

Figure 4. Two configurations of passive aerodynamic control system: **a)** with asymmetric cable connection, **b)** with symmetric cable connection

4.1. Equation of Motion for Passive Control System

For passive control systems shown in Fig. 4 rotation of control flaps is proportional to the pitching motion of the deck:

$$\beta = t_\beta \alpha, \qquad \gamma = t_\gamma \alpha \tag{12a, b}$$

The state space equation of motion for passive control system may be derived from (10) as

$$\begin{bmatrix} \dot{\mathbf{y}} \\ \ddot{\mathbf{y}} \\ \dot{\mathbf{y}}_a \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\mathbf{M}_c^{-1}\mathbf{K}_c & -\mathbf{M}_c^{-1}\mathbf{C}_c & \mathbf{M}_c^{-1}\mathbf{Q}_c \\ (U/b)\mathbf{E}_c & \mathbf{0} & (U/b)\mathbf{R}_b \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \\ \mathbf{y}_a \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_c^{-1}\mathbf{T}'\mathbf{P}_{buff} \\ \mathbf{0} \end{bmatrix} \tag{13a}$$

with the matrices given as

$$\mathbf{M}_c = \mathbf{T}'\mathbf{M}\mathbf{T},\ \mathbf{C}_c = \mathbf{T}'\mathbf{C}\mathbf{T},\ \mathbf{K}_c = \mathbf{T}'\mathbf{K}\mathbf{T},\ \mathbf{Q}_c = \mathbf{T}'\mathbf{Q},\ \mathbf{E}_c = \mathbf{E}\mathbf{T} \tag{13b-f}$$

The matrix $\mathbf{T}$ in (13) defines relationship between controlled system displacement vector $\mathbf{y}' = \lfloor h/b \quad \alpha \rfloor$ and uncontrolled system displacement vector $\mathbf{x}$:

$$\mathbf{x} = \mathbf{T}\mathbf{y}, \qquad \mathbf{T}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & t_\beta & t_\gamma \end{bmatrix} \tag{14a, b}$$

4.2. Numerical Simulations of Passive Control Systems

The system with asymmetric cable connection (Fig. 4a) is studied for rotations of leading and trailing flap assumed to be equal:

$$\beta = \gamma = t\alpha \tag{15}$$

where t is a positive gain of the control.

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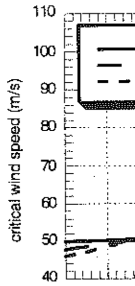

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Applying control law (15) to deck-flaps system and performing eigenvalue analysis of state-space equation of motion (13) yields poles of the controlled system. Critical wind speed for the system with asymmetric cable connection versus control gain for flaps of width $0.5m$, $1.75m$, and $3.0m$ is shown in Fig. 5a. It may be seen that efficiency of the system for flutter suppression increases with increasing the width of the flaps. The highest critical wind speed for control gain range under consideration is obtained for flaps of width $3.0m$ and equals $110m/s$ (improvement of 120%).

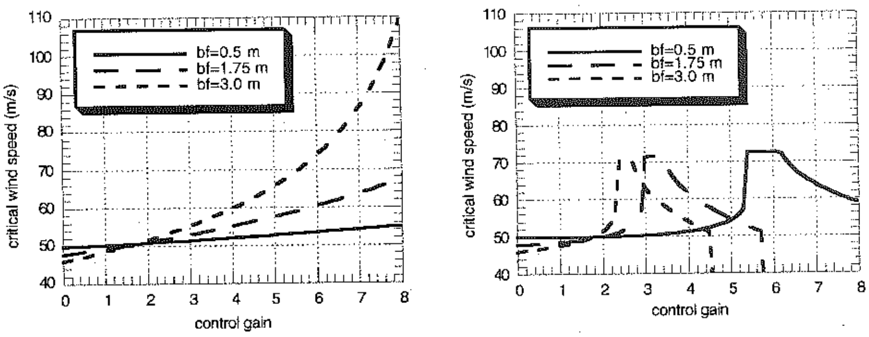

Figure 5. Variation of critical wind speed of passive aerodynamic control: **a)** for the system with asymmetric cable connection, **b)** for the system with symmetric cable connection

The system with symmetric cable connection is also studied with rotations of leading and trailing flap equal in magnitude. Motion of control surfaces is in this case govern by the following equations:

$$\beta = t\alpha. \qquad \gamma = -t\alpha \tag{16a, b}$$

for positive gains of the control t.

Critical wind speed for the system with symmetric cable connection versus control gain for flaps of width $0.5m$, $1.75m$, and $3.0m$is shown in Fig. 5b. The maximum critical wind speed is $72.5m/s$, $71.5m/s$, and $70.5m/s$, respectively. The control action cannot effect divergence of the bridge and this type of instability poses restriction on the maximum improvement in critical wind speed. The maximum improvement in critical wind speed does not depend on the size of control surfaces. For relatively large flaps of width $3.0m$ stabilizing action may be attributed mainly to the forces generated on the flaps. However, for very small flaps of width $0.5m$ the forces generated on the flaps are small in amplitude and hence, stabilizing action might be attributed mainly to the changes in flow pattern about the deck introduced by motion of control surfaces.

The maximum critical wind speed for the system with symmetric cable connection is much smaller than for the system asymmetric cable connection. Nevertheless, it provides an increase of 45%, which can be sufficient for most cases. Main advantage of the system is, however, its simplicity, coming from the independence from the wind direction. The system may use efficiently relatively small flaps, which can be light and of a simple structural design. Hence, flaps of width $0.5m$ are recommended.

5. CONCLUSIONS

In this paper the deck-flaps control system consisting of additional control surfaces attached directly to the bridge deck is proposed. Motion of the control surfaces is related to the pitching motion of the deck through additional cables and prestressed springs. Description of unsteady aerodynamic forces is obtained from the Theodorsen solution for wing-aileron-tab combination. Time domain model of self-exited aerodynamic forces is found through rational function approximation.

Two configurations of control system are proposed. The first one can provide improvement in critical wind speed of as much as 120%. However, its configuration must be changed as wind direction changes and requires relatively large flaps. The second of the proposed systems has a configuration which does not have to be altered as wind direction changes. Furthermore, it employs flaps of small size and hence, simple design. A system with flaps of $0.5m$ increases critical wind speed of 45%.

Further study on a multi-degree of freedom full bridge model should be conducted to obtain quantitative information about the effectiveness of the proposed passive aerodynamic control of flutter.

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ABSTRACT

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1. INTRODUCT

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